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ABSTRACT

This publication is part of a series prepared in cooperation with the National Science Teachers Association (NSTA), the American Association for the Advancement of Science (AAAS), and the National Council of Teachers of Mathematics (NCTM), by the Eisenhower National Clearinghouse for Mathematics and Science Education. This document focuses on the groundbreaking activities of the Scope, Sequence and Coordination (SS&C) Project initiated by NSTA. This project is a science curriculum reform movement designed to help teachers make their science programs more interesting and relevant to student needs. The document describes the basic premises on which the SS&C project is built, the programs at various sites that have been part of the project's implementation, and the results and ongoing efforts at those sites. It also offers advice on how to implement and overcome major obstacles on an SS&C program. (MKR)

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ED 460 822

A Perspective on Reform in Mathematics and Science Education

by

The National Council of Teachers of Mathematics

for

The Eisenhower National Clearinghouse for Mathematics and Science Education

Monograph #1

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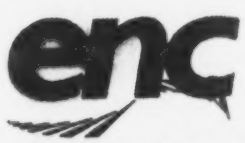

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A Perspective on Reform in Mathematics and Science Education

by

The National Council of Teachers of Mathematics
Kenneth E. Vos, Ph.D.
Editor

for
The Eisenhower National Clearinghouse for Mathematics and Science Education

Monograph #1

1996

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Foreword

A major goal of the Eisenhower National Clearinghouse for Mathematics and Science Education (ENC) is to support national reform efforts to improve teaching and learning in mathematics and science. Integral to these efforts is the leadership of the National Council of Teachers of Mathematics (NCTM), the American Association for the Advancement of Science, and the National Science Teachers Association in developing and promoting these reforms. In cooperation with these organizations, the Clearinghouse has prepared a series of publications, each entitled *A Perspective on Reform in Mathematics and Science Education*.

This publication in the series, Monograph #1, focuses on the activities of NCTM. NCTM has led in the effort both to reform mathematics in the nation's schools and to set national standards for education within several disciplines. Included is an overview of NCTM's landmark work, *Curriculum and Evaluation Standards for School Mathematics* (the *Standards*), which discusses NCTM's philosophy and provides sample lessons for all grade levels, giving teachers an opportunity to begin to interpret the *Standards* for their classrooms.

ENC is pleased to have collaborated with NCTM in producing this publication. We believe reform efforts are crucial to achieving the National Education Goal 5, "By the Year 2000, U.S. students will be first in the world in mathematics and science achievement," and we believe cooperation among the organizations reaching for that goal is imperative. We want to thank Kenneth E. Vos and Cynthia C. Rosso of NCTM and the Communications staff at Aspen Systems Corporation who were responsible for the production of this publication.

Dr. Len Simutis, Director
Eisenhower National Clearinghouse

Introduction

The National Council of Teachers of Mathematics (NCTM) organization is leading the effort for mathematics reform in our nation's schools. NCTM desires to assist the classroom teacher in interpreting how to educate most effectively all learners in the discipline of mathematics. Recently NCTM published two landmark documents, *Curriculum and Evaluation Standards for School Mathematics* (1989), and *Professional Standards for Teaching Mathematics* (1991), supporting change in mathematics education. A third document, *Assessment Standards for School Mathematics* was published in May 1995. These documents are motivating classroom teachers to reevaluate their curriculum and instructional modes in mathematics. This booklet presents a brief overview of the *Curriculum and Evaluation Standards for School Mathematics*, known as the *Standards*, for the classroom teacher. The primary focus of this booklet is on getting started with the process of interpreting the *Standards* for mathematics in your own classroom. In other words, you should try at least one aspect of the *Standards* in your classroom and build from this positive experience to more complex changes. Most teachers find that change is more likely to last if it occurs in a collegial atmosphere—where you are working with other teachers in your school to implement change—where you can collaborate.

What Is Included in This Booklet

Because the primary focus of this booklet is on getting started in the mathematics reform movement, a major portion is dedicated to actual samples of how the *Standards* can be incorporated into the mathematics classroom. Preceding the samples are brief sections devoted to what NCTM believes about mathematics learning and teaching, the history of the *Standards*, and current and future efforts involving the *Standards*. Following the samples is a short section that explains potential challenges to mathematics reform during the 1990s. At the conclusion of the booklet is an annotated bibliography of selected NCTM materials.

How To Use This Booklet

Classroom teachers can use this booklet in at least three ways.

Brute-Force Method
You could begin on page 1 and read each page word for word. This method is not recommended since the intent of the booklet is to be a resource to get started and not a definitive textbook on mathematics reform.

Stop-and-Look Method
You could start reading in one of the beginning sections to sample the content, and then browse through other sections.

Busy-Teacher Method
You could begin by reading the "This is what we believe" section to familiarize yourself with the basic assumptions of the *NCTM Standards*. You could then review the sample standards and activities appropriate for your grade level(s), plan how you would revise the activity for your classroom, and try the revised activity with learners. Later, you could review the selected resources from NCTM and read the remaining sections of the booklet. The important thing is to take subsequent action.

Note:
(1) This booklet will use extensive passages from *Curriculum and Evaluation Standards for School Mathematics* or its Executive Summary. Because this booklet is a summary of the *Standards*, these direct quotes will not be noted. Direct quotes from other sources will be cited.
(2) Permission is granted to photocopy material from this booklet for educational purposes.

Invitation to Teachers

"The fastest growing jobs require much higher math, language, and reasoning capabilities than current jobs, while slowly growing jobs require less."—*Workforce 2000 Everybody Counts,* p. 2.

Our society is entering a new era—the Information Age—a period in which information is the raw material and communication its means of production. The transition from an industrial to an informational society can be attributed to the increased availability of affordable technology, including computers and calculators. Technology is changing the workplace, the home, and daily life at a pace that is exhilarating to some, and frightening to others.

The effects of technological innovation on business, government, and industry are paralleled by dramatic changes in the physical, social, and life sciences. More than many other areas of study and application, mathematics is being taken in new directions. Modern technology has precipitated a shift in what mathematics a person needs to know. At the same time, as computers open frontiers of ideas once beyond exploration, new mathematics is being created as technological applications emerge. Mathematics now offers unprecedented potential for helping people understand the world.

Yet, in the midst of a technological revolution, the teaching of mathematics has remained relatively unchanged. As it has for centuries, the learning of mathematics has often relied on rote memorization of rules. Many teachers continue to view success in mathematics as an immediate mastery of facts and rules.

To equip students for productive, fulfilling lives in the Information Age, the definition of success in mathematics—the objective of mathematics—must be transformed. In the decade leading to the 21st century, school mathematics must be redefined to address the current and future needs of our students and Nation. The National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (the *Standards*) is the mathematical sciences community's answer to these needs. The *Standards* outlines learning outcomes in mathematics appropriate for tomorrow's schools.

The fundamental objective of education always has been to prepare students for life. The new objectives for school mathematics—inspired by the critical needs of our society—do not depart from this tradition but, rather, reaffirm it.

Need: An informed electorate

In a democratic country in which political and social decisions involve increasingly complex technical issues, an educated, informed electorate is critical. Current issues such as environmental protection, nuclear energy, defense spending, medical advances, space exploration, and taxation involve many interrelated questions; their thoughtful resolution requires technological knowledge and understanding. In particular, citizens must be able to read and interpret complex and sometimes conflicting information. This need cannot be met as long as society allows 25 percent of high school students to drop out before graduation and only a few of the remaining students to complete 4 years of high school mathematics.

Need: Mathematically literate workers

A mathematically literate workforce is crucial because advancing methods of production make ever-higher demands on workers' knowledge and skills. Traditional standards for basic mathematical competence are not sufficient. Workers now must understand the complexities and technologies of communication and must be prepared to ask the right questions, assimilate new information, solve unfamiliar problems in unconventional ways, and work cooperatively as well as independently.

"Mathematical expectations for new employees in industry:

- The ability to set up problems with the appropriate operations.
- Knowledge of a variety of techniques to approach and work on problems.
- Understanding of the underlying mathematical features of a problem.
- The ability to work with others on a problem.
- The ability to see the applicability of mathematical ideas to common and complex problems.
- Preparation for open problem situations, since most real problems are not well formulated.
- Belief in the utility and value of mathematics."

—Henry Pollak
Curriculum and Evaluation Standards for School Mathematics, p. 4.

Need: Opportunity for all students

Current statistics indicate that most people who study advanced mathematics are white males. Because women and most minorities study less mathematics, they are seriously underrepresented in careers involving science and technology. As a result, their economic standard of living is often lower and less secure. In such careers, mathematics has been a filter that often

screens out women and minority group members. Thus, besides being an issue of justice, creation of a society where equal opportunity exists in access to mathematics has become an economic necessity.

Need: Problem-solving skills that serve lifelong learning

In the next 25 years, the typical worker will change jobs four or five times. Throughout their lives, individuals will need to be adaptable—to continue to explore the world, accommodate changing conditions, and actively seek and create new knowledge. This need for flexibility implies that mathematics education must emphasize a dynamic literacy that is centered on the application of mathematics to problem solving.

In sum, the overall objective of teaching mathematics should be to develop in each and every student an understanding of mathematics that lasts a lifetime and grows to meet changing demands. The curriculum and evaluation standards developed by NCTM provide a framework for accomplishing that objective.

You are invited to read this booklet. Share it with your colleagues. Become involved at a level appropriate for you. No matter what your involvement, your support for improvement of mathematics education will add to the momentum for change.

Be a part of working to make mathematical power a reality for every student.
Professional Standards for Teaching Mathematics, p. 194.

In reality, no one can teach mathematics. Effective teachers are those who can stimulate students to learn mathematics.
Everybody Counts, p. 58.

What We Believe

A standard is a statement that can be used to judge the quality of a mathematics curriculum or methods of evaluation.
Curriculum and Evaluation Standards for School Mathematics, p. 2.

Mathematics is a science of **pattern and order.**
Everybody Counts, p. 31.

The *Standards* are statements about what is valued in mathematics education. Not merely a list of outcomes or a scope-and-sequence of a mathematics program, the *Standards* is a new vision of school mathematics.

Several underlying assumptions shape the vision of mathematics as set forth in the *Standards*.

Mathematics is something a person does.

Knowing mathematics means being able to use it in purposeful ways. To learn mathematics, students must be engaged in exploring, conjecturing, and thinking, rather than only in rote learning of rules and procedures. Mathematics learning is not a spectator sport. When students construct personal knowledge derived from meaningful experiences, they are much more likely to retain and use what they have learned. This fact underlies the new role of teachers in providing experiences that help students make sense of mathematics, to view and use it as a tool for reasoning and problem solving.

Mathematics has broad content encompassing many fields.

Mathematics is more powerful than ever before because yesterday's arithmetic, geometry, algebra, and statistics are being enhanced by new results from applied fields. From the early grades onward, students can benefit from exposure to a broad range of content that reveals the usefulness of mathematics.

Through this wide-ranging exposure, students can build a foundation of understanding to support their study and use of mathematics in a technological society.

Mathematics instruction and learning can be improved through appropriate evaluation.

Evaluation should be an integral part of teaching and learning mathematics. Evaluation should concentrate on assessing what students know, how they think, and how they feel about mathematics. This turns program evaluation toward the goal of ensuring that all students are successful in learning mathematics.

Mathematical power can—and must—be at the command of all students in a technological society.

With proper instruction, students can gain the necessary confidence, knowledge, and techniques for applying mathematics to everyday problems. Students can develop strategies for continuing to learn and use mathematics with confidence as the world about them continually changes.

The *Standards* expresses the desirability of providing all students with opportunities to share the new vision of mathematics and to learn in ways consistent with it. Students should be encouraged and enabled to explore, reason logically, draw inferences, and employ a variety of mathematical methods to become mathematically literate and capable of developing their mathematical power.

Educational goals for students must reflect the importance of mathematical literacy. Toward this end, the K–12 standards articulate five general goals for all students:

(1) That they learn to value mathematics.

(2) That they become confident in their ability to do mathematics.

(3) That they become mathematical problem solvers.

(4) That they learn to communicate mathematically.

(5) That they learn to reason mathematically.

> Learning should engage students both intellectually and physically. They must become active learners, challenged to apply their prior knowledge and experience in new and increasingly more difficult situations.
> *Curriculum and Evaluation Standards for School Mathematics,* p. 67.

These goals imply that students should be exposed to numerous and varied interrelated experiences that encourage them to value the mathematical enterprise, to develop mathematical habits of mind, and to understand and appreciate the role of mathematics in human affairs; that they should be encouraged to explore, to guess, and even to make and correct errors so that they gain confidence in their ability to solve complex problems; that they should read, write, and discuss mathematics; and that they should conjecture, test, and build arguments about a conjecture's validity.

The opportunity for all students to experience these components of mathematical training is at the heart of our vision of a quality mathematics program. The curriculum should be permeated with these goals and experiences so that they become commonplace in the lives of students. We are convinced that if students are exposed to the kinds of experiences outlined in the *Standards*, they will gain *mathematical power.* This term denotes an individual's abilities to explore, conjecture, and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve nonroutine problems. This notion is based on the recognition of mathematics as more than a collection of concepts and skills to be mastered; it includes methods of investigating and reasoning, means of communication, and notions of context. In addition, for each individual mathematical power involves the development of personal self-confidence.

Toward this end, we see classrooms as places where interesting problems are regularly explored using important mathematical ideas. Our premise is that *what* a student learns depends to a great degree on *how* he or she has learned it. For example, one could expect to see students recording measurements of real objects, collecting information and describing their properties using statistics, and exploring the properties of a function by examining its graph. This vision sees students studying much of the same mathematics currently taught, but with quite a different emphasis; it also sees some mathematics being taught that in the past has received little emphasis in schools.

1. Learning to value mathematics.
Students require numerous and varied experiences related to the cultural, historical, and scientific evolution of mathematics so that they may appreciate the role of mathematics in the development of our contemporary society and explore relationships among mathematics and the disciplines it serves—the physical and life sciences, the social sciences, and the humanities.

Throughout the history of mathematics, practical problems and theoretical pursuits have stimulated one another to such an extent that it is impossible to disentangle them. Even today as theoretical mathematics has burgeoned in its diversity and deepened in its complexity and abstraction, it has become more concrete and vital to our technologically oriented society. It is the intent of this goal—learning to value mathematics—to focus attention on the need for student awareness of the interaction between mathematics and the historical situations from which it has developed and the impact that interaction has on our culture and our lives.

2. Becoming confident in one's own ability.
As a result of studying mathematics, students need to view themselves as capable of using their growing mathematical power to make sense of new problem situations in the world around them. To some extent, everyone is a mathematician and does mathematics consciously. To buy at the market, to measure a strip of wallpaper, or to decorate a ceramic pot with a regular pattern is doing mathematics. School mathematics must endow all students with a realization that doing mathematics is a common human activity. Having numerous and varied experiences allows students to trust their own mathematical thinking.

> **A** curriculum is an operational plan for instruction that details what mathematics students need to know, how students are to achieve the identified curricular goals, what teachers are to do to help students develop their mathematical knowledge, and the context in which learning and teaching occur. *Curriculum and Evaluation Standards for School Mathematics,* p. 1.
>
> **I**nstructional approaches should engage students in the process of learning rather than transmit information for them to receive. *Curriculum and Evaluation Standards for School Mathematics,* p. 67.

3. Becoming a mathematical problem solver.
The development of each student's ability to solve problems is essential if he or she is to be a productive citizen. We strongly endorse the first recommendation made in *An Agenda for Action* [National Council of Teachers of Mathematics 1980]: "Problem solving must be the focus of school mathematics" [p. 2]. To develop such abilities, students need to work on problems that may take hours, days, and even weeks to solve. Although some may be relatively simple exercises to be accomplished independently, others should involve small groups or an entire

class working cooperatively. Some problems also should be open-ended with no "right" answer, and others need to be formulated.

4. Learning to communicate mathematically.
The development of a student's power to use mathematics involves learning its signs, symbols, and terms. This is best accomplished in problem situations where students have an opportunity to read, write, and discuss ideas in which use of the language of mathematics becomes natural. As students communicate their ideas, they learn to clarify, refine, and consolidate their thinking.

5. Learning to reason mathematically.
Making conjectures, gathering evidence, and building an argument to support such notions are fundamental to doing mathematics. In fact, demonstrations of good reasoning should be rewarded even more than students' abilities to find correct answers.

In summary, the intent of these goals is that students will become mathematically literate. This term denotes an individual's ability to explore, to conjecture, and to reason logically, as well as to use a variety of mathematical methods effectively to solve problems. By becoming literate, their mathematical power should develop.

History and Information on the *Standards*

Contrary to the fears of many, the availability of calculators and computers has expanded students' capability of performing calculations. There is no evidence to suggest that the availability of calculators makes students dependent on them for simple calculations.
Curriculum and Evaluation Standards for School Mathematics, p. 8.

Since 1920, the National Council of Teachers of Mathematics (NCTM) has been dedicated to the improvement of school mathematics instruction at all levels. Through its many publications, conferences, and other services, this professional organization of more than 110,000 members provides a forum for discussing new developments, sharing innovative classroom experiences, and evaluating trends in the teaching of mathematics.

In 1986, the Board of Directors of the National Council of Teachers of Mathematics established the Commission on Standards for School Mathematics as one means to help improve the quality of school mathematics. NCTM took a first step toward preparing today's students for meeting the challenges of tomorrow's society by charging the Commission with two tasks:

- Create a coherent vision of what it means to be mathematically literate in a world that relies on calculators and computers to carry out mathematical procedures, and in a world where mathematics is rapidly growing and being extensively applied in diverse fields.

- Create a set of standards to guide revision of the school mathematics curriculum and associated evaluation toward this vision.

The *Standards* were drafted during the summer of 1987 and revised during the summer of 1988 by the members of four Working Groups, each representing a cross-section of mathematics educators, including classroom teachers, supervisors, educational researchers, teacher educators, and university mathematicians.

> The Commission, as well as the writing groups, was most concerned that the final product represent a consensus of the mathematical sciences education community on what students in American schools should know and be able to do as a result of their study of school mathematics. They were further interested in developing a plan for school mathematics that was both realistic and achievable in the nation's schools. Above all, it had to be a document that 'made sense' of school mathematics for the reader [Dossey 1989, p. 667].
>
> On Tuesday, 21 March 1989, the NCTM held a formal press conference in the Willard Hotel in Washington, D.C., to release the *Standards*. Thomas Romberg, John Dossey, and Shirley Frye were joined on the podium by Sally K. Ride of NASA and Stanford University and by Bud C. Wonsiewicz, vice-president of US WEST Advanced Technologies, Englewood, Colorado. The latter two confirmed the significance of the *Standards* by detailing the importance of quality mathematics education to our nation and its social, economic, and strategic future [Dossey 1989, p. 668].

The *Standards* offer a framework for curriculum development—a logical network of relationships among identified topics of study. This network accommodates the varying needs of local schools because numerous paths might be taken through it.

Although the *Standards* specify the key elements of a high-quality school mathematics program, they neither list topics for particular grades nor show a "scope-and-sequence" chart. Instead, the 40 curriculum standards divide the discussion of curriculum content into three grade-level groups: K–4, 5–8, and 9–12. Individual standards include discussion of the need to accommodate students' differing talents, abilities, interests, achievements, and needs.

The 14 evaluation standards fall into three categories. The first three standards pertain to **assessment strategies** for the curriculum *Standards*. The next seven provide teachers with information that will help them improve instruction. This cluster of evaluation standards closely parallels several topics of the curriculum

> If we want students to investigate, explore, and discover, assessment must not measure just mimicry mathematics.
> *Everybody Counts*, p. 70.

standards: problem solving, communication, reasoning, and mathematical concepts and procedures. In addition, the evaluation standards speak to students' "mathematical disposition"—their attitudes about mathematics and inclinations to think and act in positive ways toward mathematics. Teachers can use these standards to make comprehensive assessments of students' mathematical progress. The final four evaluation standards describe the information that teachers, administrators, and policymakers need when assessing a mathematics program's quality and effectiveness of instruction.

The first three curriculum standards for each grade level and three of the evaluation standards deal with problem solving, communication, and reasoning. A fourth curriculum standard, "Mathematical Connections," is predicated on the belief that, despite the necessity of teaching specific concepts and procedures, mathematics must be approached as a unified whole. Its concepts, procedures, and intellectual processes are so interrelated that, in a significant sense, its "whole is greater than the sum of its parts." Consequently, curricula should deliberately include instructional activities to reveal the connections among ideas and procedures related to mathematics and its applications in other subject matter content areas.

"Mathematics as Problem Solving" emphasizes students' use of strategies to:

- Investigate and understand mathematical content.
- Recognize and formulate problems.
- Solve a wide variety of problems.
- Generalize solutions and strategies, applying them to new problems.

- Acquire confidence in their ability to use mathematics meaningfully.

"Mathematics as Communication" focuses on students' development of using language and symbols to:

- Clarify thinking about mathematical ideas and situations.
- Express mathematical ideas orally, in writing, and with physical material, pictures, and diagrams.
- Understand and value mathematical notation's role.
- Realize that representing, discussing, listening, writing, and reading mathematics are vital aspects of mathematics study and use.

"Mathematics as Reasoning" concentrates on leading students to:

- Draw logical conclusions about mathematics.
- Use mathematical models, facts, properties, and relationships to explain their thinking.
- Justify solution-finding processes and answers.
- Use patterns and relationships to analyze mathematical situations.

> Students retain best the mathematics that they learn by processes of internal construction and experience.
> *Everybody Counts*, p. 59.

- Make and evaluate mathematical conjectures.
- Believe that mathematics makes sense.

For each grade-level group, nine or 10 content standards supplement the first four curriculum standards. Some standards share similar titles because some content areas need emphasis across the curriculum. However, the concepts and processes vary by level. Others emphasize specific content that needs to be developed at a particular level.

Curriculum Standards

Curriculum Standards for Grades K–4

The standards for grades K–4 are based on the following assumptions:

- The K–4 curriculum should be conceptually oriented.
- The K–4 curriculum should actively involve children in doing mathematics.
- The K–4 curriculum should emphasize the development of children's mathematical thinking and reasoning abilities.
- The K–4 curriculum should emphasize the application of mathematics.
- The K–4 curriculum should include a broad range of content.
- The K–4 curriculum should make appropriate and ongoing use of calculators and computers.

K–4 Standards

1. Mathematics as Problem Solving.
2. Mathematics as Communication.
3. Mathematics as Reasoning.
4. Mathematical Connections.
5. Estimation.
6. Number Sense and Numeration.
7. Concepts of Whole Number Operations.
8. Whole Number Computation.
9. Geometry and Spatial Sense.
10. Measurement.
11. Statistics and Probability.
12. Fractions and Decimals.
13. Patterns and Relationships.

These 13 K–4 standards emphasize aspects of curriculum and instruction that depart from current practice. The following aspects are to receive increased and decreased attention.

Increased Attention in K–4 Mathematics

Number

- Number sense.
- Place-value concepts.
- Estimation of quantities.

Operations and Computation

- Meaning of operations.
- Operation sense.
- Mental computation.
- Estimation and the reasonableness of answers.
- Selection of an appropriate computational method.
- Use of calculators for complex computation.
- Thinking strategies for basic facts.

Geometry and Measurement

- Properties of geometric figures.
- Geometric relationships.
- Spatial sense.
- Process of measuring.
- Concepts related to units of measurement.
- Actual measuring.
- Estimation of measurements.
- Use of measurement and geometry ideas throughout curriculum.

Probability and Statistics

- Collection and organization of data.
- Exploration of chance.

Patterns and Relationships

- Pattern recognition and description.
- Use of variables to express relationships.

Problem Solving

- Word problems with a variety of structures.
- Use of everyday problems.
- Applications.
- Study of patterns and relationships.
- Problem-solving strategies.

Instructional Practices

- Use of manipulative materials.
- Cooperative work.
- Discussion of mathematics.
- Questioning.
- Justification of thinking.
- Writing about mathematics.
- Problem-solving approach to instruction.
- Use of calculators and computers.

Decreased Attention in K–4 Mathematics

Number

- Early attention to reading, writing, and ordering numbers symbolically.

Operations and Computation

- Complex paper-and-pencil computations.
- Isolated treatment of paper-and-pencil computations.
- Addition and subtraction without renaming.
- Isolated treatment of division facts.
- Long division.
- Long division without remainders.
- Paper-and-pencil fraction computation.
- Use of rounding to estimate.

Geometry and Measurement

- Primary focus on naming geometric figures.
- Memorization of equivalencies between units of measurement.

Problem Solving

- Use of clue words to determine which operation to use.

Instructional Practices

- Rote practice.
- Rote memorization of rules.
- One answer and one method.
- Use of worksheets.
- Written practice.
- Teaching by telling.

Curriculum Standards for Grades 5–8

The standards for grades 5–8 are based on the following assumptions:

- Problem situations that establish the need for new ideas and motivate students should serve as the context for mathematics.
- Communication with and about mathematics and mathematical reasoning should permeate the curriculum.
- A broad range of topics should be taught, including number concepts, computation, estimation, functions, algebra, statistics, probability, geometry, and measurement.
- Technology, including calculators, computers, and videos, should be used when appropriate.
- Each classroom should be equipped with ample sets of manipulative materials and supplies.
- Teachers and students should have access to appropriate resource materials from which to develop problems and ideas for exploration.
- All students should have access to a calculator.
- Each classroom should have at least one computer available at all times for demonstrations and student use.

Grades 5–8 Standards

1. Mathematics as Problem Solving.
2. Mathematics as Communication.
3. Mathematics as Reasoning.
4. Mathematical Connections.
5. Number and Number Relationships.
6. Number Systems and Number Theory.
7. Computation and Estimation.
8. Patterns and Functions.
9. Algebra.
10. Statistics.
11. Probability.
12. Geometry.
13. Measurement.

These 13 standards emphasize aspects of curriculum and instruction that depart from current practice. The following aspects are to receive increased and decreased attention.

Increased Attention in 5–8 Mathematics

Problem Solving

- Pursuing open-ended problems and extended problem-solving projects.
- Investigating and formulating questions from problem situations.
- Representing situations verbally, numerically, graphically, geometrically, or symbolically.

Communication

- Discussing, writing, reading, and listening to mathematical ideas.

Reasoning

- Reasoning in spatial contexts.
- Reasoning with proportions.
- Reasoning from graphs.
- Reasoning inductively and deductively.

Connections

- Connecting mathematics to other subjects and to the world outside the classroom.
- Connecting topics within mathematics.
- Applying mathematics.

Number/Operations/Computation

- Developing number sense.
- Developing operation sense.
- Creating algorithms and procedures.
- Using estimation both in solving problems and in checking the reasonableness of results.
- Exploring relationships among representations of, and operations on, whole numbers, fractions, decimals, integers, and rational numbers.
- Developing an understanding of ratio, proportion, and percent.

Patterns and Functions

- Identifying and using functional relationships.
- Developing and using tables, graphs, and rules to describe situations.
- Interpreting among different mathematical representations.

Algebra

- Developing an understanding of variables, expressions, and equations.
- Using a variety of methods to solve linear equations and informally investigate inequalities and nonlinear equations.

Statistics

- Using statistical methods to describe, analyze, evaluate, and make decisions.

Probability

- Creating experimental and theoretical models of situations involving probabilities.

Geometry

- Developing an understanding of geometric objects and relationships.
- Using geometry in solving problems.

Measurement

- Estimating and using measurement to solve problems.

Instructional Practices

- Actively involving students individually and in groups in exploring, conjecturing, analyzing, and applying mathematics in both a mathematical and a real-world context.
- Using appropriate technology for computation and exploration.
- Using concrete materials.
- Being a facilitator of learning.
- Assessing learning as an integral part of instruction.

Decreased Attention in 5–8 Mathematics

Problem Solving

- Practicing routine, one-step problems.
- Practicing problems categorized by types (e.g., coin, age, rate).

Communication

- Doing fill-in-the-blank worksheets.
- Answering questions that require only yes, no, or a number as responses.

Reasoning

- Relying on outside authority (teacher or answer key).

Connections

- Learning isolated topics.
- Developing skills out of context.

Number/Operations/Computation

- Memorizing rules and algorithms.
- Practicing tedious paper-and-pencil computations.
- Finding exact forms of answers.
- Memorizing procedures, such as cross-multiplication, without understanding.
- Practicing rounding numbers out of context.

Algebra

- Manipulating symbols.
- Memorizing procedures and drilling on equation solving.

Statistics

- Memorizing formulas.

Probability

- Memorizing formulas.

Geometry

- Memorizing geometric vocabulary.
- Memorizing facts and relationships.

Measurement

- Memorizing and manipulating formulas.
- Converting within and between measurement systems.

Instructional Practices

- Teaching computations out of context.
- Drilling on paper-and-pencil algorithms.
- Teaching topics in isolation.
- Stressing memorization.
- Teacher being the dispenser of knowledge.
- Testing for the sole purpose of assigning grades.

Curriculum Standards for Grades 9–12

The standards for grades 9–12 are based on the following assumptions:

- Students entering grade 9 will have experienced mathematics in the context of the broad, rich curriculum outlined in the K–8 standards.
- The level of computational proficiency suggested in the K–8 standards will be expected of all students; however, no student will be denied access to the study of mathematics in grades 9–12 because of a lack of computational facility.
- Although arithmetic computation will not be a direct object of study in grades 9–12, conceptual and procedural understandings of number, numeration, and operations and the ability to make estimations and approximations and to judge the reasonableness of results will be strengthened in the context of applications and problem solving, including those situations dealing with issues of scientific computation.
- Scientific calculators with graphing capabilities will be available to all students at all times.
- A computer will be available at all times in every classroom for demonstration purposes, and all students will have access to computers for individual and group work.
- At least 3 years of mathematical study will be required of all secondary school students.
- These 3 years of mathematical study will revolve around a core curriculum differentiated by the depth and breadth of the treatment of topics and by the nature of applications.
- Four years of mathematical study will be required of all students intending to go to college.
- These 4 years of mathematical study will revolve around a broadened curriculum that includes extensions of the core topics and for which calculus is no longer viewed as the capstone experience.
- All students will study appropriate mathematics during their senior year.

Grades 9–12 Standards

1. Mathematics as Problem Solving.
2. Mathematics as Communication.
3. Mathematics as Reasoning.
4. Mathematical Connections.
5. Algebra.
6. Functions.
7. Geometry from a Synthetic Perspective.
8. Geometry from an Algebraic Perspective.
9. Trigonometry.
10. Statistics.
11. Probability.
12. Discrete Mathematics.

13. Conceptual Underpinnings of Calculus.

14. Mathematical Structure.

These 14 standards emphasize aspects of curriculum and instruction that depart from current practice. The following aspects are to receive increased and decreased attention.

Increased Attention in 9–12 Mathematics

Algebra

- Use of real-world problems to motivate and apply theory.
- Use of computer utilities to develop conceptual understanding.
- Computer-based methods such as successive approximations and graphing utilities for solving equations and inequalities.
- Structure of number systems.
- Matrices and their applications.

Geometry

- Integration across topics at all grade levels.
- Coordinate and transformation approaches.
- Development of short sequences of theorems.
- Deductive arguments expressed orally and in sentence or paragraph form.
- Computer-based explorations of 2–D and 3–D figures.
- Three-dimensional geometry.
- Real-world applications and modeling.

Trigonometry

- Use of appropriate scientific calculators.
- Realistic applications and modeling.
- Connections among the right triangle ratios, trigonometric functions, and circular functions.
- Use of graphing utilities for solving equations and inequalities.

Functions

- Integration across topics at all grade levels.
- Connections among a problem situation, its model as a function in symbolic form, and the graph of that function.
- Function equations expressed in standardized form as checks on the reasonableness of graphs produced by graphing utilities.
- Functions that are constructed as models of real-world problems.

Statistics

- Use of statistical analysis.

Probability

- Investigation of probability models.

Discrete Mathematics

- Exploration of discrete mathematics.

Instructional Practices

- Active involvement of students in constructing and applying mathematical ideas.
- Problem solving as a means as well as a goal of instruction.
- Effective questioning techniques that promote student interaction.
- Use of a variety of instructional formats (small groups, individual explorations, peer instruction, whole-class discussions, project work).
- Use of calculators and computers as tools for learning and doing mathematics.
- Student communication of mathematical ideas orally and in writing.
- Establishment and application of the interrelatedness of mathematical topics.
- Systematic maintenance of student learnings and embedding review in the context of new topics and problem situations.
- Assessment of learning as an integral part of instruction.

Decreased Attention in 9–12 Mathematics

Algebra

- Word problems by type, such as coin, digit, and work.
- Simplification of radical expressions.
- Use of factoring to solve equations and to simplify rational expressions.
- Operations with rational expressions.
- Paper-and-pencil graphing of equations by point plotting.
- Logarithm calculations using tables and interpolation.
- Solution of systems of equations using determinants.
- Conic sections.

Geometry

- Euclidean geometry as a complete axiomatic system.
- Proofs of incidence and betweenness theorems.
- Geometry from a synthetic viewpoint.
- Two-column proofs.
- Inscribed and circumscribed polygons.
- Theorems for circles involving segment ratios.
- Analytic geometry as a separate course.

Trigonometry

- Verification of complex identities.
- Numerical applications of sum, difference, double-angle, and half-angle identities.
- Calculations using tables and interpolation.
- Paper-and-pencil solutions of trigonometric equations.

Functions

- Paper-and-pencil evaluation.
- Graphing of functions by hand using tables of values.
- Formulas given as models of real-world problems.
- Expression of function equations in standardized form in order to graph them.
- Treatment as a separate course.

Instructional Practices

- Teacher and text as exclusive sources of knowledge.
- Rote memorization of facts and procedures.
- Extended periods of individual seatwork practicing routine tasks.
- Instruction by teacher exposition.
- Paper-and-pencil manipulative skill work.
- Relegation of testing to an adjunct role with the sole purpose of assigning grades.

Evaluation Standards

Standards for evaluation are based on the following assumptions:

1. Student assessment must be integral to instruction.

> Assessment should be an integral part of teaching. It is the mechanism whereby teachers can learn how students think about mathematics as well as what students are able to accomplish. *Everybody Counts,* p. 69.

2. Multiple means of assessment methods must be used.

3. All aspects of mathematical knowledge and its connections must be assessed.

4. Instruction and curriculum must be considered equally in judging the quality of a program.

General Assessment

1. Alignment.
2. Multiple Sources of Information.
3. Appropriate Assessment Methods and Uses.

Student Assessment

4. Mathematical Power.
5. Problem Solving.
6. Communication.
7. Reasoning.
8. Mathematical Concepts.
9. Mathematical Procedures.
10. Mathematical Disposition.

Program Evaluation

11. Indicators for Program Evaluation.
12. Curriculum and Instructional Resources.

> By confusing means and ends, by making testing more important than learning, present practice holds today's students hostage to yesterday's mistakes.
> *Everybody Counts*, p. 70.

13. Instruction.
14. Evaluation Team.

The 14 evaluation standards emphasize aspects of assessment and program evaluation that depart from current practice. The following aspects are to receive increased and decreased attention.

Increased Attention for Evaluation

- Assessing what students know and how they think about mathematics.
- Having assessment be an integral part of teaching.
- Focusing on a broad range of mathematical tasks and taking a holistic view of mathematics.
- Developing problem situations that require application of a number of mathematical ideas.
- Using multiple assessment techniques, including written, oral, and demonstration formats.
- Using calculators, computers, and manipulatives in assessment.
- Evaluating the program by systematically collecting information on outcomes, curriculum, and instruction.
- Using standardized achievement tests as only one of many indicators of program outcomes.

Decreased Attention for Evaluation

- Assessing what students do not know.
- Having assessment be simply counting correct answers on tests for the sole purpose of assigning grades.
- Focusing on a large number of specific and isolated skills organized by a content-behavior matrix.
- Using exercises or word problems requiring only one or two skills.
- Using only written tests.
- Excluding calculators, computers, and manipulatives from the assessment process.
- Evaluating the program only on the basis of test scores.
- Using standardized achievement tests as the only indicator of program outcomes.

The *Standards* is a document designed to establish a broad framework to guide reform in school mathematics in the next decade. In it, a vision is given of what the mathematics curriculum should include in terms of content priority and emphasis. The challenge we issue to all who are interested in the quality of school mathematics is to work collaboratively to use these curriculum and evaluation standards as the basis for change so that the teaching and learning of mathematics in our schools is improved.

Current Status of the *Standards*

> **M**athematics instruction must not reinforce the common impression that the only problems amenable to mathematical ideas are those that have unique correct answers.
> *Everybody Counts*, p. 44.

Many people—especially educators, parents, business and industry leaders, engineers, scientists, and mathematicians—already recognize that school mathematics reform is imperative. As individuals and members of organizations, they are eager to take action and move forward. As a result, the *Standards* has been influential: Several states used it in writing their state curriculum frameworks in mathematics and in setting criteria for textbook adoption, many school districts are using it to review their mathematics curriculum, some textbook publishers are using it as a planning guide, and some testing programs have modified their assessments to incorporate its suggestions.

In 1992, NCTM began a study of the implementation of the *Standards*. Preliminary data from a pilot study show that the majority of the teachers agreed that:

- Manipulative materials help many students understand mathematics.
- Inclusion of real-life applications does not disrupt the flow of instruction.
- Virtually all students can learn to think mathematically [NCTM 1992, p. 3].

However, contrary to the fundamental message of the *Standards,* many teachers surveyed still think that:

- Students need to master computation before going on to algebra.
- Most students need to perform complex computations with speed and accuracy.
- The primary purpose of mathematics is to prepare students for further study in mathematics [NCTM 1992, p. 3].

Reports from teachers reveal that three types of activities recommended in the *Standards* are frequently used by a substantial number of classes at each grade level (K–4, 5–8, 9–12):

- Working in small cooperative groups.
- Learning through real-life applications.
- Making conjectures and exploring possible methods to solve a mathematics problem [NCTM 1992, p. 10].

In summary, the teachers surveyed were supportive of the notion that manipulatives assist the understanding of mathematics, that applications are appropriate in the mathematics curriculum, and that virtually all students can learn mathematics.

In 1991, NCTM published a complementary document to the *Standards*, the *Professional Standards for Teaching Mathematics*. This document is an overview of the vision of the direction mathematics instruction should take in the 1990s and beyond.

Teaching envisioned in the *Professional Standards for Teaching Mathematics* is significantly different from what many teachers themselves have experienced as students in mathematics classes. Woven into the fabric of these standards are five major shifts in the environment of mathematics classrooms that are needed to move from current practice to mathematics teaching that will empower students. Accordingly, we need to:

- Shift toward classrooms as mathematical communities and away from classrooms as simply a collection of individuals.

Because teaching mathematics well is a complex endeavor, it cannot be reduced to a recipe for helping students learn. Instead, good teaching depends on a host of considerations and understandings. *Professional Standards for Teaching Mathematics,* p. 22.

Students' learning of mathematics is enhanced in a learning environment that is built as a community of people collaborating to make sense of mathematical ideas. *Professional Standards for Teaching Mathematics,* p. 58.

These teaching standards are not intended to be an exhaustive checklist of specific concepts, skills, and behaviors that teachers must have. Instead, these standards are a set of principles accompanied by illustrations or indicators that can be used to judge what is valuable and appropriate. They give direction for moving toward excellence in teaching mathematics. *Professional Standards for Teaching Mathematics,* p. 7.

- Shift toward logic and mathematical evidence as verification and away from the teacher as the sole authority for right answers.
- Shift toward mathematical reasoning and away from mere memorization of procedures.
- Shift toward conjecturing, inventing, and problem solving and away from merely emphasizing finding the correct answer.
- Shift toward connecting mathematics, its ideas, and its applications and away from treating mathematics as a body of isolated concepts and skills.

Underlying the new vision of *Professional Standards for Teaching Mathematics* are two fundamental assumptions:

1. Teachers are the key to changing the way in which mathematics is taught and learned.

If teachers are to create a learning environment that empowers students, they need the time and resources to develop the professional teaching skills envisioned. Teachers must have ongoing professional development opportunities as well as instructional and assessment materials that are consistent with the *Standards* and the latitude to use them flexibly.

2. Teachers must have long-term support and adequate resources.

This new vision of teaching mathematics requires that teachers be supported, encouraged, and rewarded by administrators, parents, and the community at large. Although this recognition and collective support will take time to develop, it is a principal element of effecting change in the classroom environment. Change is difficult; it takes time and reliable, systematic support [NCTM 1991].

> It is a key function of the teacher to develop and nurture students' abilities to learn with and from others—to clarify definitions and terms to one another, consider one another's ideas and solutions, and argue together about the validity of alternative approaches and answers. *Professional Standards for Teaching Mathematics*, p. 58.

Immediately following publication of *Curriculum and Evaluation Standards for School Mathematics* and *Professional Standards for Teaching Mathematics*, NCTM undertook the task of showing how to put the new vision of mathematics instruction into practice to recharge teaching and learning. A series of books, called the *Addenda Series*, was written, field-tested, and published by NCTM. The books are written for three grade level groups: K–6, 5–8, and 9–12. The themes of problem solving, reasoning, communication, connections, technology, and evaluation have been woven throughout the materials. Each book in the *Addenda Series* features specially designed activities that have been field tested by classroom teachers and teacher notes in the page margins that suggest questions to assess student learning, relate the activity to a *Standards* principle, or provide insight into a teaching technique. The *Addenda Series* materials are used in the Getting Started section of this booklet. The books in the series are listed in the Annotated Bibliography of Selected NCTM Materials.

Plans for the Future

> The most important goals for mathematical learning cannot be atomized into tiny morsels of knowledge. *Everybody Counts*, p. 69.

NCTM is continuing its efforts to document the complex nature of mathematics reform in our nation's schools. A major 5-year research initiative called "The Recognizing and Recording Reform in Mathematics Education (R3M) Project" is currently measuring the breadth and depth of the vision of mathematics in a variety of school sites [Ferrini-Mundy 1993].

NCTM is also extending the scope of its present documents on standards to include assessment standards. This document is "to present teachers of mathematics with a vision of assessment that is consistent with the earlier *Standards* documents ... and to align this vision of assessment with different educational programs [NCTM 1993, p. 1]." These standards are now available and were reviewed by members of the mathematics education community. The final version proposes the following assessment standards:

- Important Mathematics—Assessment should reflect the mathematics that all students need to know and be able to do.
- Enhanced Learning—Assessment should enhance mathematics learning.
- Equity—Assessment should promote equity by giving each student optimal opportunities to demonstrate mathematical power and by helping each student meet the profession's high expectations.

- Openness—Assessment should be an open process.
- Valid Inferences—Assessment should promote valid inferences about mathematics learning.
- Consistency—Assessment should be a coherent process.

NCTM is committed to the new vision of mathematics instruction for all students. A continued focus of all efforts to promote high-quality mathematics teaching and learning is a primary goal of the National Council of Teachers of Mathematics.

Simply put, assessment should not rely on a single instrument or technique.
Curriculum and Evaluation Standards for School Mathematics, p. 192.

As we need standards for curricula, so we need standards for assessment. We must ensure that tests measure what is of value, not just what is easy to test.
Everybody Counts, p. 70.

Getting Started

This section of the booklet attempts to present an overview of some of the standards and appropriate activities that reflect the vision of the *Standards*. Each sample includes the complete statement of the standard, teacher notes for the appropriate activity, *Addenda Series* source, and a copy or outline of the activity. Although an activity may include more than one aspect of more than one standard, only one standard is highlighted for each sample activity. Each sample activity is targeted to a grade level in order to include activities at the different levels within the K–12 grades. However, as is true with many mathematics topics, some activities can be adapted for instruction with students younger or older than the targeted grade level.

Primary

Standard 6—Number Sense and Numeration

In grades K–4, the mathematics curriculum should include whole number concepts and skills so that students can:

- Construct number meanings through real-world experience and the use of physical materials.
- Understand our numeration system by relating counting, grouping, and place-value concepts.
- Develop number sense.
- Interpret the multiple uses of numbers encountered in the real world [NCTM 1989, p. 38].

Domino Fun—Level: 1st Grade

Purpose:
The purpose of this activity is to have students develop an understanding of number relationships and of ordering numbers.

Materials:
A few sets of dominoes; each student should have a copy of the Domino Record Sheet.

Background:
The teacher should introduce the idea of dominoes to the total group of students. Have the students tell about dominoes and what the dots mean. The teacher will ask a series of questions before the Domino Record Sheet is given to each student.

Source:
NCTM *Addenda Series, First-grade book,* pp. 8–9, 11.

Beginning

Hold up a domino and have the students tell the total number of dots. Then ask them to find a domino that has more dots and to find another that has fewer dots.

Hold up a domino and ask the students to find a domino that has one dot more and to find another that has one dot less.

Teacher Questions (TQ): *Did all of you find the same domino? How are they all the same?*

TQ: *Is there a domino for which you cannot find another domino with more dots? With fewer dots?*

TQ: *I am thinking of a domino that has five dots. What could it look like?*

TQ: *How many dots would the domino have if it had one more dot? What would it look like? Is there more than one answer?*

TQ: *I have another domino. If you added one more dot, there would be eight dots altogether. What domino could I have?*

Draw the possibilities as students suggest the various dominoes that have seven dots.

Give the students a copy of the Domino Record Sheet. Demonstrate how to draw a domino and record the solutions. Notice that the Domino Record Sheet, with your modification, can be used in a variety of ways, some very structured and others more open ended.

Domino Record Sheet

My domino	Total dots	One less	One more	My domino	Total dots	One less	One more
	____	____	____		____	____	____
	____	____	____		____	____	____
	____	____	____		____	____	____
	____	____	____		____	____	____
	____	____	____		____	____	____

Standard 9—Geometry and Spatial Sense *(Primary)*

In grades K–4, the mathematics curriculum should include two- and three-dimensional geometry so that students can:

- Describe, model, draw, and classify shapes.
- Investigate and predict the results of combining, subdividing, and changing shapes.
- Develop spatial sense.
- Relate geometric ideas to number and measurement ideas.
- Recognize and appreciate geometry in their world [NCTM 1989, p. 48].

Covering a Region—Level: 2nd Grade

Purpose:
The purpose of this activity is to have students cover a region completely with a set of congruent figures.

Materials:
Dot paper, pattern blocks or a pattern for a square, and a copy of the Cover and Count activity sheet.

Background:
To understand the concept of area, students need experience in covering a specific region. Using a variety of figures to cover a region helps students understand that different units may be used and that the number used to define the area depends on the unit. This activity shows that there is a strong connection among geometry, spatial sense, and measurement.

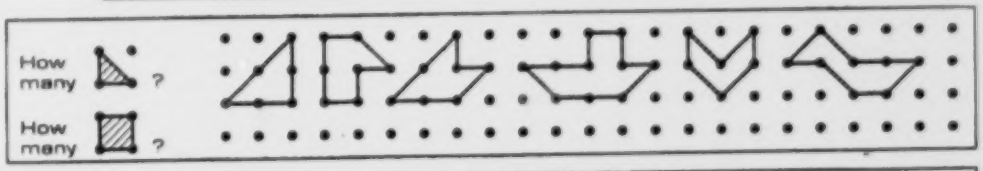
Give the students three squares (pattern blocks or cut-outs), and ask them to make as many different figures as they can by joining the squares edge to edge. They should produce two unique figures. Now try with four squares. Ask them to draw on dot paper the figures they get. Have a student show on the overhead the five unique figures. This activity allows the students to compare figures in different orientations and, through discussion, to determine which figures are congruent—the same size and shape.

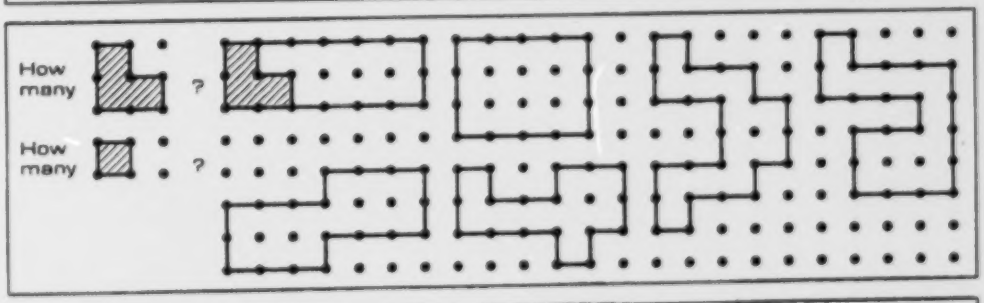
Allow the students to repeat the activity, using three or four triangles, hexagons, rhombi, or trapezoids from the pattern blocks to make as many different figures as they can.

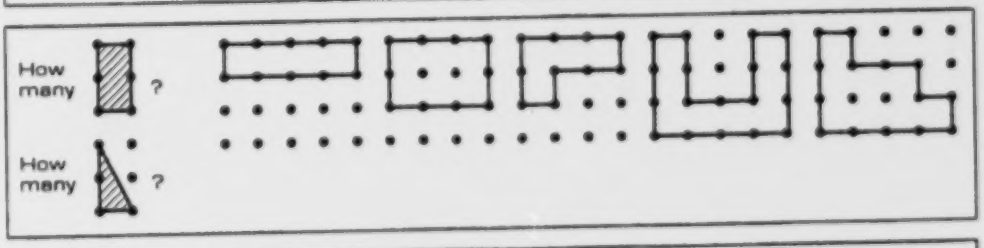
Have the students complete the Cover and Count activity sheet.

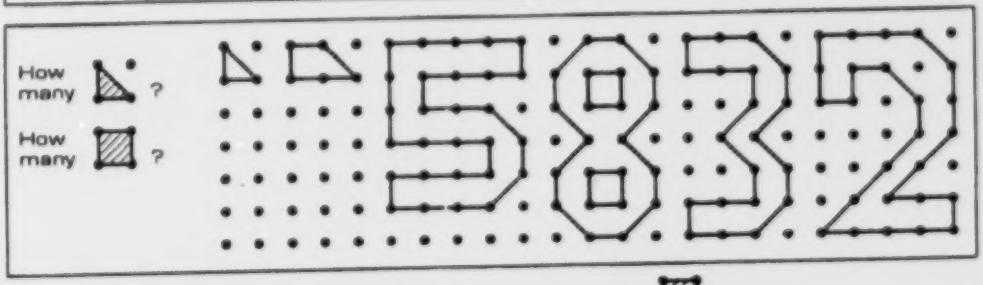
Source:
NCTM *Addenda Series, Second-grade book*, pp. 30–32.

COVER AND COUNT









Make a figure on dot paper that can be covered by exactly 8

Make a figure on dot paper that can be covered by exactly 15

Standard 13—Patterns and Relationships *(Primary)*

In grades K–4, the mathematics curriculum should include the study of patterns and relationships so that students can:

- Recognize, describe, extend, and create a wide variety of patterns.
- Represent and describe mathematical relationships.
- Explore the use of variables and open sentences to express relationships [NCTM 1989, p. 60].

Calculator Sequence—Level: 3rd Grade

Purpose:
The purpose of this activity is to have students reproduce and analyze arithmetic sequences by using a calculator.

Materials:
Paper, pencil, and at least one calculator for each group of two or three students.

Background:
During this activity the students will be repeatedly adding numbers. They can simply add the same number (e.g., 2 [+] 2 [=] 4 [+] 2 [=] 6 [+] 2 [=] 8). You should, however, have them use the arithmetic constant capability for addition that most calculators possess. You can test your calculator by pressing

2 [+] [=] [=] [=] [=] ... or 0 [+] 2 [=] [=] [=] [=] ... or 2 [+] [+] [+] [+] .... If one of these sequences produces the desired successive display of 2, 4, 6, 8, 10, ..., your calculator has an arithmetic constant.

Source:
NCTM *Addenda Series, Third-grade book*, pp. 6–7.

Calculator Sequence

Have the students key in any number on their calculators, say 17. Select any 1-digit number, say 6, and have them skip count with their calculators and record the sequence as it appears in the display. For example, 17 [+] 6 [=] [=] [=] ... Have the students note the digits in the ones place.

Teacher Questions (TQ): *Will 137 be in sequence? How would you find out?*

Such activities lay the foundation for multiplication and division algorithms. Practice skip counting by other numbers on the calculator and record the sequence. Have the students record the first five numbers of each sequence on the overhead and challenge other students to find the next number of the sequence. Have the students explain how they determined each successive number.

Write a sequence of five numbers, such as 27, 41, 55, 69, and 83 on the overhead.

TQ: *Did these numbers result from skip counting? How do you know?*

Write another number, such as 181, and ask whether or not this number is in the sequence of numbers. Show the students how to reproduce this sequence of numbers on their calculators:

27 [+] 14 [=] [=] [=] [=]

Repeatedly pressing the [=] key causes the sequence to appear on the display. Counting the number of times the [=] key is pressed helps keep track of where in the sequence 181 falls. The 12th number in the sequence is 181. Give further practice in creating a sequence on the calculator; for example, 18 [+] 5 [=] [=] [=] ... gives this sequence: 18, 23, 28, 33, 38, ...

TQ: *Is 222 in the sequence 11, 22, 33, 44, ...? What is the seventh term? The 17th term? How do you know?*

TQ: *Look at this sequence: 51, 61, 71, 81, 91, ... Is 123 one of the numbers in this sequence? 1111? 910? Explain.*

TQ: *Reproduce this sequence on the calculator: 13, 94, 175, 256, 337, ... The number 1552 is in this sequence. What term in the sequence is 1552?*

TQ: *Use the following clues to reproduce the sequence on the calculator. The fourth number in the sequence is 95. Each number in the sequence is 12 more than the previous number. What is the first number? [59] The 20th number? [287]*

TQ: *Play the following game in which you share one calculator with your partner. Start at 50. In turn, subtract any 1-digit number except 0. The first player to reach 0 wins. Can you find a strategy to win? Does it matter who goes first? Suppose we change the rules and decide that the person who first reaches 0 loses; how might this change your strategy to win? Repeat the game but start with different numbers (e.g., 91, 131, 288).*

Intermediate

Standard 8—Whole Number Computation

In grades K–4, the mathematics curriculum should develop whole number computation so that students can:

- Model, explain, and develop reasonable proficiency with basic facts and algorithms.
- Use a variety of mental computation and estimation techniques.
- Use calculators in appropriate computational situations.
- Select and use computation techniques appropriate to specific problems and determine whether the results are reasonable. [NCTM 1989, p. 44]

The Answer Is...Level: 4th Grade

Purpose:
The purpose of this activity is to give students mental arithmetic opportunities.

Materials:
Display paper or overhead and a calculator if appropriate.

Background:
This activity could be used as a way to turn waiting time into learning time. The activity requires few materials, and it can be presented in oral form very quickly.

Understanding multiple representations for numbers is a crucial precursor to solving many of the problems students encounter. Making conjectures, gathering evidence, and building an argument to support ideas are fundamental to doing mathematics.

Source:
NCTM *Addenda Series, Fourth-grade book*, pp. 13–14.

The Answer Is ...

Announce a number and ask each student in turn to state a question for which that number is the answer. Give the students several examples to encourage nonroutine thinking.

Teacher Questions (TQ): *The answer is 36. What is 3 x 12? What is 3 x 2 x 6? How much money is one quarter, one dime, and one penny? What is six more than the number of days in November? What is the number of inches in 3 feet?*

Make a large display for each answer and the questions it generated.

Extension

Change the focus from giving many descriptions of a known number to discovering an unknown number, "the answer," through a series of progressive clues. For example, if the "answer" is 7, you might give the students these clues:

I am less than 10.

I am an odd number.

I am more than 6.

I am a prime number.

Show the clues one at a time and encourage the students to give possible answers after each clue is revealed.

After you model several of these "What number am I?" puzzles, have the students write their own for others to solve. You may wish to give them guidelines, such as the riddles must have at least four clues but no more than seven clues, or the riddles should have two possible answers.

Try some different clues to build mental arithmetic skills.

Standard 10—Statistics *(Intermediate)*

In grades 5–8, the mathematics curriculum should include exploration of statistics in real-world situations so that students can:

- Systematically collect, organize, and describe data.
- Construct, read, and interpret tables, charts, and graphs.
- Make inferences and convincing arguments that are based on data analysis.
- Evaluate arguments that are based on data analysis.
- Develop an appreciation for statistical methods as a powerful means for decisionmaking [NCTM 1989, p. 105].

A Random Walk—Level: 5th Grade

Purpose:
The purpose of this activity is to have students use a random number table to generate data. Students also will develop an intuitive notion about adding positive and negative numbers and some beginning ideas about probability.

Materials:
Copy of A Random Walk activity sheet for each group of two students and a transparency of the activity sheet.

Background:
Students should work in pairs, one reading the numbers from the random number table and the other making the steps on the number line.

Usually when we take a walk we know where we are going, but sometimes we just meander. A random walk, like the ones that students will take in this activity, is determined by chance. Tell the students that they are going to investigate such a walk on a number line.

Each step of this walk is specified by a number [1–6] in a random number table. For each walk, a pair of students will choose six adjacent numbers in a row or a column, beginning at any place in the table. These six numbers specify the six steps; an even number will indicate a move to the right and an odd number will mean a move to the left. For example, if the first two numbers in the selected row were 4 and 1, the first step would be a move of 4 to the right from 0 and the second step would be a move of 1 to the left from the end of the first step. The final position would be R3.

Use the transparency of A Random Walk and practice walking on the number line several times with the class before having the students collect the data from 10 walks.

Teacher Questions (TQ): *About how many times out of 100 will a walk stay on this number line?*

TQ: *How many steps will most walks take before going off?*

TQ: *On which step will most walks go off this number line?*

TQ: *Is a walk more likely to go off at the right or at the left? Why?*

TQ: *Will more walks end to the left or to the right of 0?*

After talking about these predictions, let each student pair take 10 walks, recording where they land on the number line or recording which step during their walk ended off this number line. This information should be recorded in the "right off" or "left off" columns.

Discuss how the results of the student pairs compare with the predictions they made. Combine the data for the entire class.

Source:
NCTM *Addenda Series, Fifth-grade book*, pp. 20–23.

A RANDOM WALK

1. To determine a walk, chose any six contiguous numbers in a line or column on the Random Number Table. These six numbers give the six steps of the walk.

2. On the number line, begin at 0. Move the number of units specified for each step. If the number is odd, move left. If the number is even, move right.

3. Take six steps and record where you land in the table below.

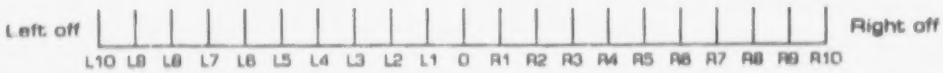
4. For the next walk, choose six numbers in any row or column.

5. Take ten walks, each of six steps.

RANDOM NUMBER TABLE

3	1	1	6	3	3	4	2	6	6	5	4
6	4	3	5	6	1	5	4	4	2	4	3
5	6	1	5	4	1	3	2	2	6	1	1
1	5	6	2	2	1	4	4	1	5	3	2
5	3	6	1	2	5	1	2	6	3	3	1
5	2	1	6	5	1	4	2	3	1	5	3
2	3	5	6	2	4	4	2	5	2	3	2
4	6	4	1	6	4	6	4	4	4	3	6
5	4	4	6	4	2	4	1	2	5	2	1
5	1	6	4	4	3	1	6	5	2	3	1
3	3	6	1	1	2	6	2	1	4	4	3
5	2	5	5	1	1	5	6	3	4	4	6

Walk one: show your steps.



Use this number line for the rest of your walks. If you want to show all steps, draw your own number line.

Record where you landed on each of the ten walks.

Left off	L10	L9	L8	L7	L6	L5	L4	L3	L2	L1	0	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	Right off

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Standard 4—Mathematical Connections *(Intermediate)*

In grades 5–8, the mathematics curriculum should include the investigation of mathematical connections so that students can:

- See mathematics as an integrated whole.
- Explore problems and describe results using graphical, numerical, physical, algebraic, and verbal mathematical models or representations.
- Use a mathematical idea to further their understanding of other mathematical ideas.
- Apply mathematical thinking and modeling to solve problems that arise in other disciplines, such as art, music, psychology, science, and business.
- Value the role of mathematics in our culture and society [NCTM 1989, p. 84].

Figurate Numbers—Level: 6th Grade

Purpose:
The purpose of this activity is to illustrate how the language of geometry can be used to deal with number theory questions involving odds and evens.

Material:
A copy of the Figurate (Polygonal) Numbers activity sheet for each student.

Background:
This activity, Figurate (Polygonal) Numbers, allows students to informally "prove" rules by using geometric representations. The patterns are not immediately evident to many students. Teachers should guide students' careful exploration of each number pattern. It may be helpful to separate the activity sheet, Figurate (Polygonal) Numbers, into five sections: oblong numbers, triangular numbers, square numbers, relationships among these numbers, and the optional section, pyramidal numbers.

It may also be effective to explore the real-world examples of each of these numbers. Finding examples in the school or surrounding buildings will reinforce the connections of mathematics with other areas.

Source:
NCTM *Addenda Series, Geometry in the middle grades,* pp. 14–16, 34.

ACTIVITY 5A
FIGURATE (POLYGONAL) NUMBERS

Examine the patterns and complete each table below.

1. Oblong numbers:
   G = number of dots

	1	2	3	4	5	6	... 10 ...	n
G	2	6	12	20	?	?	?	?

How can you predict the next oblong number? What patterns can you find? Describe.

2. Triangular numbers:
   T = number of dots

	1	2	3	4	5	6	... 10 ...	n
T	1	3	6	10	?	?	?	?

How can you predict the next triangular number? What patterns can you find? Describe.

3. Square numbers:
   S = number of dots

	1	2	3	4	5	6	... 10 ...	n
S	1	4	9	16	?	?	?	?

How can you predict the next square number? What patterns can you find? Describe.

4. Can you find a relationship between the oblong and the triangular numbers? Describe it. How would you explain the relationship by using only the dot representations?

5. What relationships can you find between two consecutive triangular numbers and a square number? Describe. How would you explain these relationships by only using the dot representations?

6. Optional: Sometimes you will see fruit or cans in grocery stores piled high in the form of a pyramid as shown. The number of pieces of fruit or cans is sometimes called a *pyramidal number*.

   Pyramidal numbers:
   Y = number of dots

	1	2	3	4	... 6
Y	1	4	10	?	?

What patterns can you find? Describe.

Middle/Junior High

Standard 1—Mathematics as Problem Solving

In grades 5–8, the mathematics curriculum should include numerous and varied experiences with problem solving as a method of inquiry and application so that students can:

- Use problem-solving approaches to investigate and understand mathematical content.
- Formulate problems from situations within and outside mathematics.
- Develop and apply a variety of strategies to solve problems, with emphasis on multistep and nonroutine problems.
- Verify and interpret results with respect to the original problem situation.
- Generalize solutions and strategies to new problem situations.
- Acquire confidence in using mathematics meaningfully [NCTM 1989, p. 75].

Rumors—Level: Middle School

Purpose:
The purpose of this activity is to explore exponents in a problem-solving situation.

Materials:
A copy of the Rumors activity sheet for each student and a calculator capable of generating exponent expressions.

Background:
The problem should be posed in the context of a school setting. Make sure students understand how the rumor is being spread. An effective way to explain how the rumor is spread is to act out the problem for a few days. Allow the students to explore the problem in small groups. One way to organize the data is to construct a 3-column chart with the following headings: day, number of new people who hear the rumor on a given day, total number of people who have heard the rumor on a given day, including Tara. From the information in the chart a graph can be drawn with axes as exponent (n = day) and number of people (2n). This graph will display an exponential growth pattern. Many extensions, into exponents, graphs, and functions, can lead from the Rumors situation.

Source:
NCTM *Addenda Series, Patterns and functions, grades 5–8*, pp. 6–12.

Rumors

In Bugsville, U.S.A., at Swat Middle School, Tara, a 7th-grade student decides to start a rumor that the town of Bugsville is going to declare September 14 as National Bug Day and close all schools for the day. She tells two students the rumor with instructions that each student is to repeat the rumor to two more students the next day, and that each of these new students is to repeat the rumor to two more students on the third day, and so on. For example, on the first day, two students know the rumor; on the second day, four more students will know the rumor; on the third day, eight more students will know the rumor, and so on.

How many new students will be told the rumor on day 10?

If Tara starts her rumor on September 1 and 8,000 students are in the district, is it possible that all students will hear the rumor before September 14 and will stay home from school on that day? Why?

Explain all your thoughts about the power of this rumor.

Standard 7—Computation and Estimation *(Middle/Junior High)*

In grades 5–8, the mathematics curriculum should develop the concepts underlying computation and estimation in various contexts so that students can:

- Compute with whole numbers, fractions, decimals, integers, and rational numbers.
- Develop, analyze, and explain procedures for computation and techniques for estimation.
- Develop, analyze, and explain methods for solving proportions.
- Select and use an appropriate method for computing from among mental arithmetic, paper-and-pencil, calculator, and computer methods.
- Use computation, estimation, and proportions to solve problems.
- Use estimation to check the reasonableness of results [NCTM 1989, p. 94].

Four-In-A-Row—Level: Middle School

Purpose:
The purpose of this activity is to use strategies of estimation in a multiplication situation.

Materials:
A transparency of the Grid and Factor Board; only one calculator available.

Background:
In this activity, Four-In-A-Row, students choose factors to produce products adjacent to each other in the grid. Only one calculator is needed after the factors are chosen. Encourage students to make their estimates using strategies such as front-end or rounding while using the product of the units digits as an additional clue.

Choose one member of the class to be the "calculator" respondent who has the only calculator available. Divide the rest of the class into two groups. To begin, display the Grid and Factor Board. In turn, each group chooses two factors from the Factor Board. If the product of those numbers is displayed on the Grid, the group captures that cell. The first group to capture four cells in a row (vertically, horizontally, or diagonally) is the winning group.

You can generate additional activities by choosing a new listing of factors (e.g., decimal factors) and creating an appropriate grid of the possible products.

Source:
NCTM *Addenda Series, Developing number sense in the middle grades*, p. 27.

ACTIVITY 14:
ESTIMATING WHOLE NUMBER PRODUCTS

To the Teacher: In this activity, students choose factors to produce products adjacent to each other in the grid. One calculator is needed after the factors are chosen. Encourage students to make their estimates using strategies such as front-end or rounding while using the product of the units digits as an additional clue.

Generate additional games by choosing a new assortment of factors (e.g., all decimal factors) and creating an appropriate grid of the possible products.

Four-in-a-Row

Choose one member of the class to be the "calculator." Provide that student with a calculator. Divide the rest of the class into two teams. To begin, display the grid and the factor board below. In turn, each team chooses two factors from the factor board. If the product of those numbers is displayed on the grid, the team captures that cell. The first team to capture four cells in a row (vertically, horizontally, or diagonally) is the winning team.

Grid

187	1189	1769	943	697
1403	319	1219	1037	437
901	1159	323	551	2501
1007	253	1537	671	391
583	779	3233	667	451

Factor board

11	29
17	41
19	53
23	61

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Standard 12—Geometry *(Middle/Junior High)*

In grades 5–8, the mathematics curriculum should include study of the geometry of one, two, and three dimensions in a variety of situations so that students can:

- Identify, describe, compare, and classify geometric figures.
- Visualize and represent geometric figures with special attention to developing spatial sense.
- Explore transformations of geometric figures.
- Represent and solve problems using geometric models.
- Understand and apply geometric properties and relationships.
- Develop an appreciation of geometry as a means of describing the physical world [NCTM 1989, p. 112].

Tangrams and Visualization—Level: Middle School

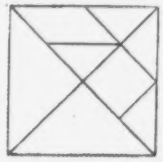
Purpose:
The purpose of this activity is for students to experience the spatial visualization of seven shapes being transformed into various configurations.

Materials:
A set of tangrams for each group, which can be cut from the given template; scissors; a copy of the Tangrams and Visualization activity sheet for each student.

Background:
This activity is rich in possibilities not only in geometry and spatial skills but also in arithmetic problem solving. Note how item nine on the activity sheet can be extended to work with fractions, decimals, and percents, thus making connections between geometry and arithmetic. The activity also furnishes some experience in motion geometry as students transform a square into various other shapes.

Source:
NCTM *Addenda Series, Geometry in the middle grades*, pp. 16, 40, 87.

ACTIVITY 6
TANGRAMS AND VISUALIZATION



1. Cut the tangram puzzle into the seven pieces.
2. Sort the pieces. Which belong together? Record your sort by making a sketch.
3. How are the five triangles alike? Different?
4. a. Put the two small triangles together so that one pair of sides of the two triangles fit together. Does this new shape match any other shape in the puzzle? Which?

   b. Repeat part (a) to make a different shape. Does this new shape match any other shape in the puzzle? Which?

   c. Repeat part (a) to make another shape. Does this new shape match any other shape in the puzzle? Which?
5. Repeat this activity by putting the two large triangles together (making one pair of sides fit together). What shapes do you get?
6. Put the three smaller triangles together to form these shapes:

   (a) a square

   (b) a rectangle that is not a square

   (c) a parallelogram that is not a rectangle

   (d) a trapezoid

   (e) a right triangle

   Sketch each result. What is the same about each figure formed in (a) through (e)?
7. Now use all seven pieces and repeat question 6. Sketch your results.
8. Create your own shape(s) with the seven pieces. Sketch your result(s).

   For example: A cat

9. a. If the puzzle were made of gold leaf and the smallest triangle cost $10, what would the whole puzzle cost? Explain how you found your answer.

   b. If the whole puzzle were worth $1, what would be the value of each piece? Explain how you found your answer.

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Secondary

Standard 2—Mathematics as Communication

In grades 9–12, the mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can:

- Reflect upon and clarify their thinking about mathematical ideas and relationships.
- Formulate mathematical definition and express generalizations discovered through investigations.
- Express mathematical ideas orally and in writing.
- Read written presentations of mathematics with understanding.
- Ask clarifying and extending questions related to mathematics they have read or heard about.
- Appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas [NCTM 1989, p. 140].

When Is a Run a Run?—Level: Secondary School

Purpose:
The purpose of this activity is to simulate outcomes of a probability event, to use simulation to estimate probabilities and expected values, and to communicate their conclusions.

Materials:
A copy of the Motivating Question for each group and coins.

Background:
Encourage students to conjecture about the Motivating Question and about how they might test their conjectures with some type of simple experiment. The majority of students will usually underestimate greatly the likelihood of long strings of occurrences in binomial outcome situations. This tendency can become part of a later discussion about the wisdom of relying on intuition in chance activities.

Have the students work in groups of three or four, encouraging them to discuss their thinking before recording individual responses to the activity questions.

Source:
NCTM *Addenda Series, A core curriculum, grades 9–12*, pp. 81–82.

When Is a Run a Run?

Motivating Question: An Obstetrician at a hospital studies recent records and finds that 1 week of 10 births shows 4 girls born in a row. She becomes curious about how likely it is in any 10 births that 4 [or more] in a row will be girls.

Group and Individual Questions

1. Write what you think would be a typical pattern for 10 births. For example, record a boy followed by two girls by writing BGG. Continue until you have 10 births recorded.

2. Repeat this exercise 4 more times until you have 5 lists of sequences for 10 births each.

3. How many of your five lists have four (or more) girls in a row?

4. On the basis of your answer to exercise 3, what is your guess about the probability that a list of 10 births contains 4 (or more) girls in a row? [If 2 of your 5 lists contained 4 (or more) girls in a row, your answer would be 2/5.]

5. Decide how you could use a coin flip to represent a birth. [Let heads represent girls and

tails, boys.] Flip a coin 10 times, recording the outcomes as a list similar to that in exercise 1.

6. Repeat this exercise 4 more times until you have 5 lists of 10 births based on coin flipping.

7. On the basis of your answer to exercise 6, what does the coin flipping suggest is an appropriate probability for four (or more) births in a row that are girls?

8. Is this a theoretical or an experimental probability? [Experimental]

Class Discussion Questions

9. Pool the class results to exercise 2 to get a class guess about the probability.

10. Pool the class results to exercise 6 to get a class experimental probability for the births of 4 (or more) girls in a row in 10 births.

11. If four girls are born in a row, we call this a run of four girls. Which are longer, the runs obtained from student guesses or the runs obtained from coin flipping? Which do you think is more like actual births, student guesses or modeling by coin tosses? Explain why you think so.

12. Given the experimental probability determined in exercise 7 for runs of 4 girls, what do you think is the probability of a run of 4 boys [or more] in 10 births?

13. The experiment using coins to represent births is called a simulation. Discuss how to set up a simulation to find the experimental probability that a family with four children will have three girls and one boy. Is order important?

Standard 7—Geometry From a Synthetic Perspective *(Secondary)*

In grades 9–12, the mathematics curriculum should include the continued study of geometry of two and three dimensions so that all students can:

- Interpret and draw three-dimensional objects.
- Represent problem situations with geometric models and apply properties of figures.
- Classify figures in terms of congruence and similarity and apply these relationships.
- Deduce properties of, and relationships between, figures from given assumptions.

In addition, college-intending students can:

- Develop an understanding of an axiomatic system through investigating and comparing various geometries [NCTM 1989, p. 157].

Collapsing Cubes—Level: Secondary School

Purpose:
The purpose of this activity is for the student to visualize solids from an expanded perspective by the motions of folding a plane figure.

Materials:
Either a set of patterns of a box without a top or squares that can be placed edge to edge to form a pattern, and a copy of the Collapsing Cubes activity sheet for each student.

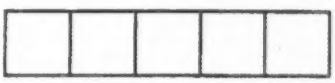
Background:
The Collapsing Cubes activity can be initiated by finding the 12 shapes possible with 5 squares. Some of the students may remember this as a search for the pentomino shapes. The challenging problem in item 4 can be initiated by finding the 35 shapes possible with 6 squares. These 35 shapes are sometimes called the hexomino shapes.

Source:
NCTM *Addenda Series, Geometry from multiple perspectives, grades 9–12*, pp. 43–44, 46.

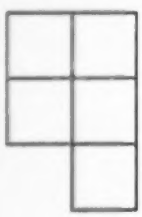
ACTIVITY 18
COLLAPSING CUBES

You can begin this investigation by trying to visualize how the sides of a cube can be collapsed into different two-dimensional patterns. First, we'll look at a simpler version of this problem. Suppose we look at a cube without one face—a box without a top. We can see that the box has only five sides and for simplicity, that these sides are all squares. If we unfold the box we might get

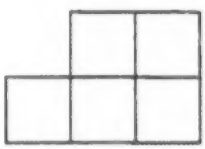
but we could not unfold the box to get the next pattern.



1. Find the other patterns that can be folded into a box without a top. Be careful not to count the same pattern twice. Said another way: rotations or reflections of a pattern are not counted a second time. For example,



is the same as



2. There are twelve different arrangements of five squares, but only the eight found in Exercise 1 can be folded up into a box without a top. Can you find all twelve? Are the areas of each of these the same? Explain! Are the perimeters of each of these the same? Explain!

3. Now see if you can collapse a regular tetrahedron in the same manner. You will have four equilateral triangles to work with. Again be careful not to count a rotation or a reflection of a combination you already have.

4. A challenging problem is to try to collapse a cube. This problem is very similar to the box-without-a-top problem. The only addition is one more square. Unfortunately, this one square increases the number of combinations to thirty-five. The number of combinations that are solutions is eleven. Find as many of them as you can. Also find as many of the six-square patterns as you can. Do these all have the same area? Perimeter?

Standard 10—Statistics *(Secondary)*

In grades 9–12, the mathematics curriculum should include the continued study of data analysis and statistics so that all students can:

- Construct and draw inferences from charts, tables, and graphs that summarize data from real-world situations.
- Use curve fitting to predict from data.
- Understand and apply measures of central tendency, variability, and correlation.
- Understand sampling and recognize its role in statistical claims.
- Design a statistical experiment to study a problem, conduct the experiment, and interpret and communicate the outcomes.
- Analyze the effects of data transformations on measures of central tendency and variability.

In addition, college-intending students can:

- Transform data to aid in data interpretation and prediction.
- Test hypotheses using appropriate statistics [NCTM 1989, p. 167].

Winning Times—Level: Secondary School

Purpose:
The purpose of this activity is to study the connections between statistics and functions by graphing points representing real-world data and modeling the observed linear relations by linear equations.

Materials:
A copy of the Winning Times activity sheet for each student and a graphing calculator if available.

Background:
The table of Winning Times is used to plot a scatter graph. Each student should draw a line that seems to best "fit" the plotted points. A line drawn in this way is called an eyeball-fit line. Later, a "best fit" line can be drawn by various methods such as the median-fit or the least squares regression techniques. By using a graphing calculator, spreadsheet, or data analysis software, other "best-fit" techniques are available. Before a fitted line is drawn, the most important question to ask is, "Could you imagine a line that represents these points?"

Using real data to teach linear equations makes the material more concrete for the student.

Source:
NCTM *Addenda Series, Data analysis and statistics, grades 9–12*, pp. 33–35, 40.

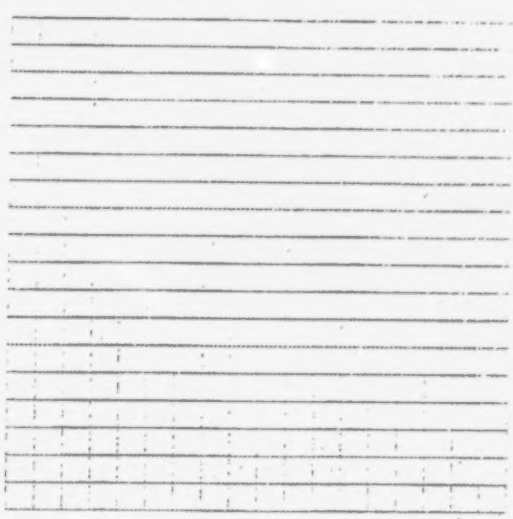
ACTIVITY 14
WINNING TIMES FOR THE WOMEN'S OLYMPIC 400-METER FREESTYLE SWIM

SHEET 1

The table lists the winning times for the women's 400-meter freestyle swim for the Olympics, 1924–1984.

Year	Time (min.:sec.)
1924	6:02.2
1928	5:42.8
1932	5:28.5
1936	5:26.4
1948	5:17.8
1952	5:12.1
1956	4:54.6
1960	4:50.6
1964	4:43.3
1968	4:31.3
1972	4:19.0
1976	4:09.9
1980	4:08.8
1984	4:07.1

Source: *World Almanac and Book of Facts*, 1992 edition, © Pharos Books 1991, New York, N.Y.



1. Using 1920 as the base year, plot (year, time).
2. Construct a best-fit line.
3. What is the slope and what does it mean? _____
4. Write the equation of the line. Use the line to predict what the times might have been if the Olympics had been held in 1940 and 1944. _____
5. Is it reasonable to use this line to predict the winning time for the 1988 Summer Games? Why or why not? _____
6. Look up the winning time for the 400-meter freestyle swim in the 1988 Summer Games and compare it to the time predicted by the best-fit line. _____

Extensions

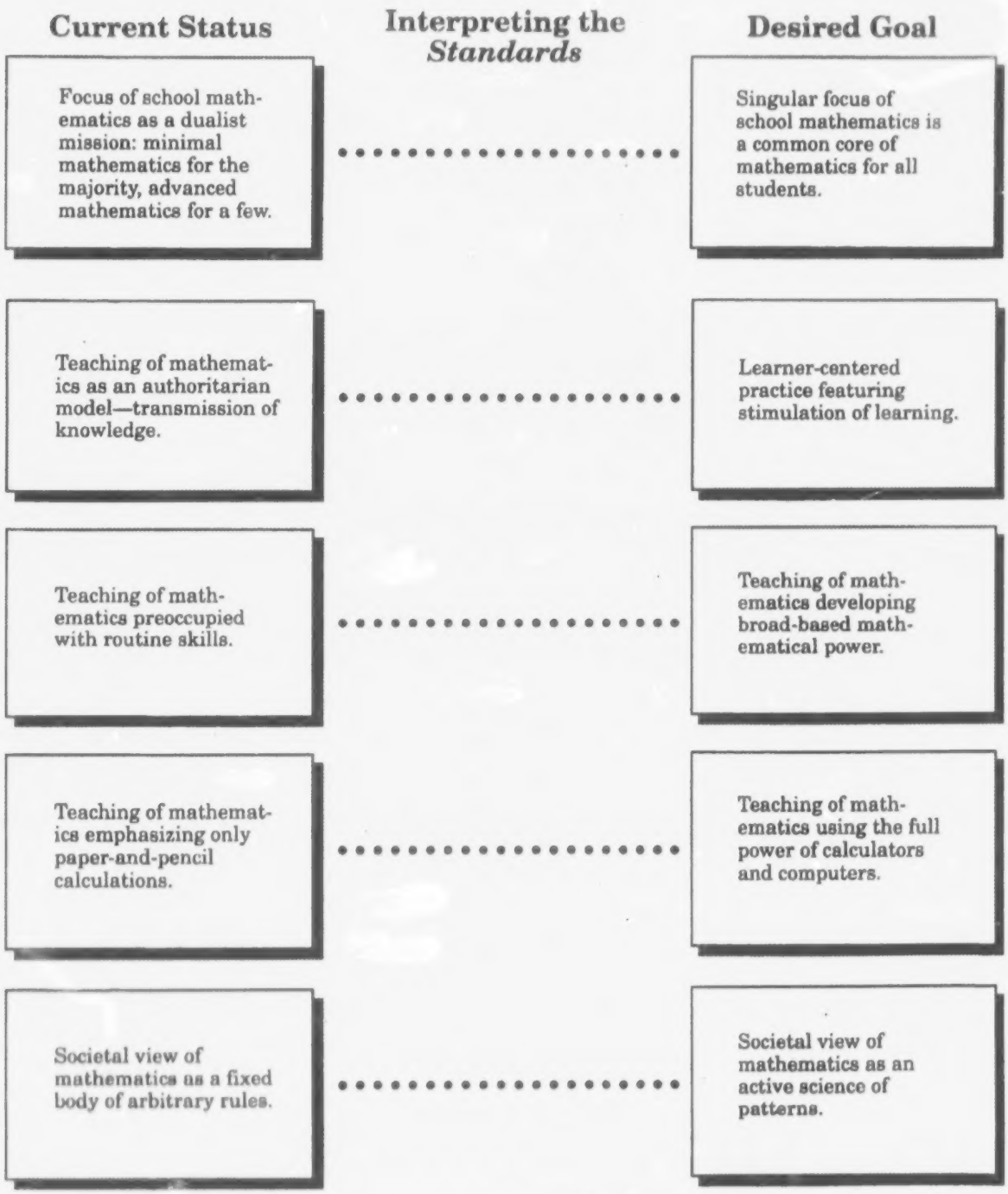
7. Use a graphing calculator to find the equation of the regression line.
8. Use a software package such as Data Insights to find the equation of the median-fit line. How do the two equations compare? Which one gives you the "best fit"? Explain your choice.

Overcoming Obstacles

> **B**oth teachers and school administrators have responsibilities to work with the community and with parents, educating them about new goals and practices in mathematics teaching. Working with parents and in the community is crucial to making change possible.
> *Professional Standards for Teaching Mathematics,* p. 15.

The steps to school mathematics reform can be viewed as insurmountable obstacles or as challenges to be met and overcome. NCTM believes that the new vision for school mathematics outlined in the *Standards* is both realistic and achievable. Content that should be in a school mathematics program has been specified. Textbooks, tests, software, manipulatives, and other teaching materials can be produced so that constructive learning will take place in classrooms. However, let there be no illusions of immediate results. What will truly transform the teaching of mathematics is the long-term commitment and coordinated effort of teachers, school administrators, school boards, parents, and all those who care about good education.

What is needed for change in mathematics instruction is a plan for transition from the current status to the desired goal. The following diagram of a possible transition is paraphrased from *Everybody Counts* [National Research Council 1989, pp. 81–84], a document contemporary with the *Standards*.


Current Status
Interpreting the *Standards*
Desired Goal
Focus of school mathematics as a dualist mission: minimal mathematics for the majority, advanced mathematics for a few.
Singular focus of school mathematics is a common core of mathematics for all students.
Teaching of mathematics as an authoritarian model—transmission of knowledge.
Learner-centered practice featuring stimulation of learning.
Teaching of mathematics preoccupied with routine skills.
Teaching of mathematics developing broad-based mathematical power.
Teaching of mathematics emphasizing only paper-and-pencil calculations.
Teaching of mathematics using the full power of calculators and computers.
Societal view of mathematics as a fixed body of arbitrary rules.
Societal view of mathematics as an active science of patterns.

Real systemic change in mathematics instruction will be a process, not an event. The changes necessary to interpret the *Standards* effectively may take at least a decade. A realistic goal would be to institute as a goal to "change" 15 percent each academic year. At this rate a complete change would be in place after 7 years. Obviously, at that point, new changes would be required. Remember, change will happen; only the rate of change can be somewhat controlled by you.

Not every school or teacher is now ready for the massive changes as envisioned in the *Standards*. However, you can begin the process by focusing on the fact that "Teachers are the key to changing the way in which mathematics is taught and learned [NCTM *Professional Standards for Teaching Mathematics*]." It is possible to make an impact on mathematics instruction by starting with small positive steps. An article in the *Arithmetic Teacher*, a publication of NCTM, listed some small steps for action. Try a few of them—be a leader in the new vision for mathematics education. Every effort made now increases the success for tomorrow's students.

> Continual change is a natural and essential characteristic of mathematics education. *Everybody Counts*, p. 73.
>
> We must be impatient enough to take action and patient enough to sustain our efforts until we see results. *Professional Standards for Teaching Mathematics*, p. 194.
>
> "We can improve. We can keep kids in school longer and achieve a better result. We cannot fail in education. If we fail, we fail our kids and we fail our future." —Jaime Escalante
> *Everybody Counts*, p. 12.

Getting Started With Some Actions

Mathematics curriculum

- Include problem-solving situations and applications in teaching wherever possible. Have students write their own problems given a situation, or have them use computational problems on a practice page to create problem situations. Use newspaper articles, advertisements, or graphs and figures as the basis on which students create problem-solving situations.
- Engage students in an active process of learning in which they create and discover mathematics concepts. Students should be encouraged to model mathematics with materials, through drawings, or through their own actions.
- Teach mathematics concepts using manipulatives and hands-on materials. Even the most primitive materials or homemade physical devices will foster the learning of abstract ideas in an appropriate setting. Plan activities that bridge from the concrete models to the pictorial stage to ensure a transfer of learning to the abstract, symbolic level.

Instruction

- Create opportunities for students to discuss mathematics and make sense of mathematics in cooperative-learning situations. As students communicate their ideas, they learn to refine their thinking, which helps to solidify their understanding. We learn best when we teach or explain ideas to someone else.

■ Offer activities that encompass various learning styles and instructional formats to stimulate learning in students of all ability levels. Give as much opportunity as possible for students to interact with you, with materials, and with each other.

■ Facilitate learning by posing questions, asking students to clarify and justify their ideas, and challenging them to seek assistance from one another. Foster an environment that stimulates learning and encourages students to conjecture, puzzle, think, reason, explore, and grapple with ideas and concepts. Classroom experiences should present students with the opportunity to apply thinking and reasoning skills.

■ Sharply reduce the number of worksheets that emphasize memorizing rules, procedures, and formulas. Create situations in which students need to analyze, evaluate, and make decisions. Include situations in which students arrive at answers in various ways and have them discuss, explain, and justify their approaches. Indicate to students that you value their thinking, not just their answers.

Communication

■ Encourage students to write in journals about mathematics. Have them communicate through drawings and illustrations, as well as in written form. This experience helps clarify their understanding of the mathematics involved.

Connections

■ Incorporate mathematics into other curricular areas to form mathematical connections. Integrate mathematics into science, social studies and current events, language arts, art, and other curricular areas. Help students see that mathematics occurs in many dimensions of living. Connect topics among and within areas of mathematics so that students do not view the topics in isolation.

Technology

■ Increase the use of such technology as calculators and computers as an integral part of mathematics instruction. Use calculators to extend thinking skills and to explore problem-solving situations rather than simply to check computations. Students should be encouraged to select the appropriate method to arrive at answers, and calculator use should be an alternative.

Assessment

■ Use various formal and informal assessment techniques. Measure mathematical understanding and power as well as achievement. Have students build portfolios that include observations, journals, group work, class presentations, and written work.

Personal Growth

■ Extend your personal growth and professional development by attending in-service workshops and professional meetings related to mathematics, reading professional journals, and sharing ideas with others [Hatfield and Price 1992, pp. 36–37].

Collectively, we as teachers have a rare opportunity to provide leadership that will make real, substantive changes in school mathematics—changes that will ensure all students a suitable and sufficient mathematical background to become productive citizens of the 21st century.

> It takes a generation to complete the mathematical education of a single individual. The first high school graduates of the next century entered elementary school in 1988. No longer can we afford to sit idly by while our children move through school without receiving mathematical preparation appropriate for the twenty-first century.
> The challenges are clear.
> The choices are before us.
> It is time to act.
> *Everybody Counts*, p. 96.

Reference List

Dossey, John A. 1989. NCTM standards for school mathematics: Visions for implementation. *Mathematics Teacher*. 82:666–68.

Ferrini-Mundy, Joan. 1993. *Interim report to the Exxon Education Foundation: The recognizing and recording of reform in mathematics education (R3M) project.* Reston: NCTM.

Hatfield, Mary M., and Jack Price. 1992. Promoting local change: Models for implementing NCTM's *Curriculum and Evaluation Standards*. *Arithmetic Teacher*. 40:34–37.

National Council of Teachers of Mathematics. 1995. *Assessment standards for school mathematics.* Reston: NCTM.

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National Research Council. 1989. *Everybody counts: A report to the nation on the future of mathematics education.* Washington, D.C.: National Academy Press.

Annotated Bibliography of Selected NCTM Materials

Standards

National Council of Teachers of Mathematics. 1991. *Professional standards for teaching mathematics.* Reston: NCTM.

Addresses professional mathematics teaching on the basis of two assumptions: teachers are key figures in changing the way mathematics is taught and learned in schools; and change requires that teachers have long-term support and adequate resources. Each strand is augmented by annotated, real-life vignettes that illustrate the recommended strategies for change.

___. 1989. *Curriculum and evaluation standards for school mathematics.* Reston: NCTM.

A document that creates a coherent vision of mathematical literacy and provides *Standards* to guide the revision of the mathematics curriculum in the next decade. The 54 *Standards* describe the curriculum in terms of content priority and emphasis in four groups—one set each for grades K–4, 5–8, and 9–12, and one set for evaluating mathematics programs and student achievement. Also available in Spanish.

Addenda Series

Grades K–6

Burton, Grace; Terrence Coburn; John Del Grande; Mary M. Lindquist; Lorna Morrow; Douglas Clements; John Firkins; and Jeane Joyner. 1991. *Kindergarten book: Addenda Series, grades K–6.* Reston: NCTM.

Focuses on patterns, number sense and operations, making sense of data, geometry, and spatial sense.

Burton, Grace; Douglas Clements; Terrence Coburn; John Del Grande; John Firkins; Jeane Joyner; Miriam A. Leiva; Mary M. Lindquist; and Lorna Morrow. 1991. *First-grade book: Addenda Series, grades K–6.* Reston: NCTM.

Explores both traditional and new topics in four areas: patterns, number sense, and operations, making sense of data, and geometry and spatial sense.

___. 1992. *Second-grade book: Addenda Series, grades K–6.* Reston: NCTM.

Shows how to encourage students to be active participants and explorers in the learning of mathematics. Filled with activities that will help second-graders advance to more abstract lessons. Margin notes supply many ideas and provide insight to teacher.

___. 1991. *Third-grade book: Addenda Series, grades K–6.* Reston: NCTM.

Offers both new and traditional topics in patterns, number sense and operations, making sense of data, and geometry and spatial sense. Features helpful margin notes that give additional information on the activities and on such topics as student self-confidence, evaluation, and grouping.

___. 1992. *Fourth-grade book: Addenda Series, grades K–6.* Reston: NCTM.

Explores a variety of problems and questions, such as the exploration of angles; the study of shadows and projective geometry; and the use of calculators to explore numerical patterns, decimal fractions, and negative numbers.

___. 1991. *Fifth-grade book: Addenda Series, grades K–6.* Reston: NCTM.

Offers investigative experiences with tessellations and similarity; the concept of variables, measurement, and estimation with large numbers; decimal estimation and computation; data interpretation; randomness and sampling; and patterns.

___. 1992. *Sixth-grade book: Addenda Series, grades K–6.* Reston: NCTM.

Features activities to investigate geometric transformations as well as solve nonroutine problems involving estimation, measurement, and numerical operations. Shows how to select appropriate statistical measures and construct graphs.

Burton, Grace. 1993. *Number sense and operations: Addenda Series, grades K–6.* Reston: NCTM.

Focuses on the importance of developing and understanding the relationships between numbers and the ability to represent numbers in a variety of ways. Also emphasizes developing a knowledge of the effects of operations and the ability to use numbers in a real-world setting.

Coburn, Terrence. 1992. *Patterns: Addenda Series, grades K–6.* Reston: NCTM.

Ideas for lessons and activities that support the fundamental importance of patterns in all areas of mathematics. Considers patterns that involve objects, number relationships, and geometric patterns.

Del Grande, John; Lorna Morrow. 1993. *Geometry and spatial sense: Addenda Series, grades K–6.* Reston: NCTM.

Shows effective ways to develop the spatial ability of children and help them explore concepts of geometry and its language. Margin notes supply questions to ask and offer guidance to the teacher.

Lindquist, Mary M. 1993. *Making sense of data: Addenda Series, grades K–6.* Reston: NCTM.

Activities illustrate dealing with data; how to gather, represent, and analyze data. Margin notes suggest questions to be asked and explain the objectives of the activities.

Grades 5–8

Curcio, Frances R.; Nadine S. Bezuk. 1994. *Understanding rational numbers and proportions: Addenda Series, grades 5–8.* Reston: NCTM.

Classroom-ready activity sheets and discussions are divided into three clusters—Exploring and Extending Rational Number Concepts, Applying Rational Number and Proportion Concepts, and Making Rational Number Connections with Similarity.

Geddes, Dorothy. 1992. *Geometry in the middle grades: Addenda Series, grades 5–8.* Reston: NCTM.

Focuses on two- and three-dimensional geometry concepts, relationships among properties of shapes, transformation geometry, and geometry-based enrichment activities. The book contains extensive notes on innovative methods of assessment and descriptions of appropriate computer technology.

Geddes, Dorothy. 1993. *Measurement in the middle grades: Addenda Series, grades 5–8.* Reston: NCTM.

Full of sample activities with a focus on learning measurement that is used in the real world. Easily reproduced activities are also geared toward strengthening students' estimation and higher order thinking skills. Part of the highly acclaimed Addenda to the Curriculum and Evaluation *Standards* for School Mathematics series. Meant to help middle school teachers bring the *Standards* into the classroom.

Phillips, Elizabeth. 1991. *Patterns and functions: Addenda Series, grades 5–8.* Reston: NCTM.

Designed for the middle grades with examples of how patterns can be used to develop or deepen understandings of important concepts in exponents, number theory, rational numbers, measurement, geometry, probability, and function.

Reys, Barbara J. 1991. *Developing number sense in the middle grades: Addenda Series, grades 5–8.* Reston: NCTM.

Discusses the description of number sense and its place in the curriculum. Includes ways teachers can develop number sense in their students, methods for evaluating number sense, activities, and an extensive resource list.

Zawojewski, Judith. 1991. *Dealing with data and chance: Addenda Series, grades 5–8.* Reston: NCTM.

Considers learner's natural understanding and daily use of data and chance and then addresses how to build on these natural abilities. Classroom activities illustrate five themes: data gathering, communication, problem solving, reasoning, and connections.

Grades 9–12

Burrill, Gail; John C. Burrill; Pamela Coffield; Gretchen Davis; Jan de Lange; Diann Resnick; and Murray Siegel. 1991. *Data analysis and statistics across the curriculum: Addenda Series, grades 9–12.* Reston: NCTM.

Examples and activities illustrate how to integrate statistical concepts into the high school mathematics curriculum. Topics include exploring and interpreting data presented in the media. Suggests how to give and grade student projects, strategies for assessment, and many practical ideas for use in the classroom.

Coxford, Arthur F., Jr. 1991. *Geometry from multiple perspectives: Addenda Series, grades 9–12.* Reston: NCTM.

Links the content proposed in the *Standards* to what is happening now in some geometry programs. Includes classroom-ready activity sheets and helpful instructional suggestions. Discusses the growing use of technology in geometry.

Froelich, Gary. 1991. *Connecting mathematics: Addenda Series, grades 9–12.* Reston: NCTM.

Classroom-ready examples illustrate how new content, such as data analysis and matrices, and new perspectives on familiar content, such as rectangular coordinates, functions, mathematical reasoning, and problem solving, can connect traditional topics that often seem isolated. Focuses throughout on connections between mathematics and real-world situations.

Heid, M. Kathleen; Jonathan Choate; Charlene Sheets; and Rose Mary Zbiek. 1995. *Algebra in a Technological World: Addenda Series, Grades 9–12.* Reston: NCTM.

Addresses the teaching and learning of high school algebra in light of the NCTM *Standards* and the dramatic changes brought about by graphing calculators and computer software. Many classroom-tested activities shift students and teachers away from paper-and-pencil exercises toward using algebraic functions and mathematical modeling to explore real-world situations. Includes activity sheets and teacher suggestions.

Meiring, Steven; Rheta Rubenstein; James E. Schultz; Jan de Lange; and Donald L. Chambers. 1992. *A core curriculum—making mathematics count for everyone: Addenda Series, grades 9–12.* Reston: NCTM.

Offers several possible curriculum models for organizing the mathematics content recommended in the *Standards*. Presents sample syllabi, lessons, and activities that help frame a curriculum to develop mathematical power. Special margin notes with suggestions on assessment techniques and on how instruction can be adapted to the thinking of students and to the resources available.

Yearbooks

Aichele, Douglas B., ed. 1994. *Professional development for teachers of mathematics.*

Helps you discover how creative mathematics teachers, administrators, and teacher educators are using the NCTM *Standards* to transform their preservice and inservice programs. Describes outstanding teacher training programs at all the K–12 grade levels—including a model project for preparing elementary school mathematics specialists. Explains how veteran teachers, in cooperation with educational and business communities, are reeducating themselves to reach the goal of a quality mathematics education for every student.

Cooney, Thomas J., ed. 1990. *Teaching and learning mathematics in the 1990s, 1990 yearbook.* Reston: NCTM.

Explores the changing role of teachers and students in a new decade of mathematics reform. Studies the relationship between research and practice; suggests effective teaching methods consistent with the *Standards*. Recommends ways of broadening student assessment and increasing the participation of women and minority groups in mathematics. Also emphasizes the importance of mathematical applications and the need to consider how students shape classroom events, the impact of technology, and the need to empower teachers as decisionmakers.

Coxford, Arthur F., Jr., ed. 1988. *The ideas of algebra, K–12, 1988 yearbook.* Reston: NCTM.

Helps you improve your classroom presentation of algebra so that every learner will experience the thrill of understanding. Areas of emphasis include the accessibility of algebraic ideas, equations and expressions, word problems, and the use of technology in the algebra classroom.

Fey, James T., ed. 1992. *Calculators in mathematics education, 1992 yearbook.* Reston: NCTM.

Includes essays on such topics as the most important ways that calculators can influence the content and the process of mathematics instruction and how calculators have influenced testing in State assessment programs and college admissions.

Kenney, Margaret, ed. 1991. *Discrete mathematics across the curriculum, K–12, 1991 yearbook.* Reston: NCTM.

Explores a topic singled out in the *Standards* as an area needing change. Looks at how discrete mathematics promotes the making of mathematical connections, provides a setting for problem solving, capitalizes on technological settings, and fosters critical thinking and mathematical reasoning. Explores the following topics: graph theory; matrices; counting methods; recursion, iteration, and induction; and algorithms.

Lindquist, Mary M., ed. 1987. *Learning and teaching geometry, K–12, 1987 yearbook.* Reston: NCTM.

Begins with perspectives on the teaching of geometry and moves on to its uses in problem solving, applications, activities, blending geometry with other areas of mathematics, and preparing teachers to teach geometry. Makes the point that there is a great body of knowledge, especially research about how students learn, that should be called on when decisions are being made about the geometry curriculum.

Shoen, Harold L., ed. 1986. *Estimation and mental computation, 1986 yearbook.* Reston: NCTM.

Fosters a broader view of estimation and its companion, mental computation. Supports estimation in various domains, and stresses the interdependency between the process of estimating in a particular area and the understanding of mathematical concepts in that area.

Trafton, Paul R., ed. 1989. *New directions for elementary school mathematics, 1989 yearbook.* Reston: NCTM.

Explores areas of profound change in elementary school mathematics, such as problem solving, communication, children's reasoning processes, and how they develop mathematical ideas. Contains practical suggestions for teachers and curriculum developers; classroom investigations of measurement, fractions, calculators, and statistics; and samples of children's work that reinforces an active approach to learning.

Webb, Norman L., ed. 1993. *Assessment in the mathematics classroom, 1993 yearbook.* Reston: NCTM.

Provides the latest ideas and procedures for assessing the teaching and learning of mathematics. It offers specific examples in three areas: the techniques of assessment, managing assessment, and the issues and perspectives in classroom assessment.

Other Selected Resources

Artzt, Alice F., and Claire M. Newman. 1990. *How to use cooperative learning in the mathematics class.* Reston: NCTM.

Discusses the reasons for using cooperative learning in the mathematics classroom, the benefits for all students, and some ways teachers can begin a cooperative learning program and make it work.

Beaumont, Vern; Roberta Curtis; and James Smart. 1986. *How to teach perimeter, area, and volume.* Reston: NCTM.

Especially written for elementary and middle school teachers. Practical suggestions for teaching the concepts of measurement.

Blume, Glendon W., and Robert F. Nicely, Jr., eds. 1991. *A guide for reviewing school mathematics programs.* Reston: NCTM.

Created to help school districts capitalize on the current climate for curriculum redesign and enhancement. Designed to enable school district

personnel to analyze their mathematics programs and to explore directions for change.

Charles, Randall; Frank Lester; and Phares O'Daffer. 1987. *How to evaluate progress in problem solving.* Reston: NCTM.

Contains ideas that are practical, sensible, and easy to implement at all levels of instruction. Helps you decide what you are trying to evaluate, describes evaluation techniques, gives tips on organizing and managing an evaluation program, and explains how to use the results.

Coburn, Terrence. 1987. *How to teach mathematics using a calculator: Activities for elementary and middle school.* Reston: NCTM.

Presents the calculator as an instructional tool to assist in the development of concepts, help reinforce skills, promote higher level thinking, and enhance problem-solving instruction. Illustrated activities, which can be duplicated for classroom use, include numeration and counting, addition and subtraction, multiplication and division, decimals, problems and applications, prealgebra, measurement, geometry, percent, and student evaluation.

Cuevas, Gilbert, and Mark Driscoll, eds. 1993. *Reaching all students with mathematics.* Reston: NCTM.

Combines a rich set of stories that range from incremental changes from the traditional approaches to daring and revolutionary efforts to widen students' participation and success in mathematics. Addresses the goal of the *Standards* to provide all students with a comprehensive mathematics education—with special attention to including those students from underrepresented groups, whose diverse backgrounds and needs have traditionally kept them marginalized.

Hirsch, Christian R., and Robert A. Laing. 1993. *Activities for active teaching and learning: Selections from the Mathematics Teacher.* Reston: NCTM.

A rich source of classroom activities for grades 7–10, organized into chapters on problem solving, numeracy, algebra and graphs, geometry and visualization, and data analysis and probability. Selected activities supporting the *Standards* engage students in the use of manipulatives, calculators, graphing calculators, and computers. Each activity includes a teacher's guide with objectives, materials, suggestions, solutions, and three or four easily reproducible activity sheets.

House, Peggy, ed. 1987. *Providing opportunities for the mathematically gifted, K–12.* Reston: NCTM.

Defines what gifted means and suggests how to identify gifted students. Explains the external forces that may affect the student and gives guidelines for establishing programs for gifted students at all grade levels. Also gives alternative programs, resources for teachers, and references.

Kuhs, Therese. Project director. 1992. *Mathematics assessment: Alternative approaches.* videotape. Reston: NCTM.

A 71-minute videotape with a printed viewer's guide. An action plan for assessment is explored by watching teachers implement the new methods with their students. The videotape discusses alternative approaches to assessment, uses classroom dramatizations, and includes an indepth commentary by a panel of mathematics educators.

Payne, Joseph N., ed. 1990. *Mathematics for the young child.* Reston: NCTM.

A book full of activities and suggestions for teacher educators and teachers in preschool through grade 4. Topics include computers for mathematics learning and examples of how problem-solving abilities and attitudes can be a central focus and how the teacher can be a planner of mathematics instruction.

Rowan, Thomas E., and Lorna Morrow, eds. 1992. *Implementing the K–8 curriculum and evaluation standards: Readings from the Arithmetic Teacher.* Reston: NCTM.

A user-friendly reference of practical ideas. This book draws from a column run in the *Arithmetic Teacher* from 1989 to 1991. Articles help put the *Standards* into practice in the elementary and middle school classrooms. Divided into five parts: (1) themes that cut across mathematics; (2) number; (3) space and dimension; (4) data collection and interpretation; and (5) patterns, relations, functions, and algebra.

Stenmark, Jean K., ed. 1992. *Mathematics assessment: Myths, models, good questions, and practical suggestions.* Reston: NCTM.

Supplies assessment models adapted from those used by other teachers, as well as step-by-step instructions on how to use portfolios and many other techniques in grades K–12. Incorporates the latest methods to evaluate your students' success to get the most useful feedback.

Swetz, Frank, and J. S. Hartzler, eds. 1991. *Mathematical modeling in the secondary school curriculum: A resource guide of classroom exercises.* Reston: NCTM.

Examines the principles and techniques of teaching mathematical modeling. Includes a selected sampling of successful classroom modeling exercises.

Thiessen, Diane, and Margaret Matthias, eds. 1992. *The wonderful world of mathematics: A critically annotated list of children's books in mathematics.* Reston: NCTM.

Reviews each book's content and accuracy, its illustrations and their appropriateness, the author's writing style, and the included activities. Rates each book for its usefulness in teaching mathematical concepts.

Welchman-Tischler, Rosamond. 1992. *How to use children's literature to teach mathematics.* Reston: NCTM.

A wonderfully illustrated book. Uses popular children's tales like "Stone Soup," "Caps for Sale," and others to help students grasp mathematics concepts such as graphing and measuring.

Journals

Teaching Children Mathematics (supersedes the Arithmetic Teacher)
This journal is published monthly from September to May. Features articles for preservice and inservice elementary school teachers and teacher educators pre-K–6. Special sections including Investigations, reproducible activity sheets for classroom use, including "at-home ideas;" A Teacher's Journal; reviews of books and instructional materials, including computer software and reports on educational research having implications for classroom practice and more.

Mathematics Teaching in the Middle School
This journal is published five times a year. Addresses the learning needs of all middle school students, the demands these needs place on their teachers, and issues that capture the vitality of mathematics and the characteristics of the middle-grades student. The journal focuses on intuitive, exploratory investigations that help students develop a strong, conceptual mathematical base. Such a foundation leads to greater mathematical abstraction, as appropriate for middle school grades.

Mathematics Teacher
This journal is published monthly from September to May. Emphasizes practical ways of helping teachers in secondary schools, 2-year colleges, and teacher education institutions to teach mathematics effectively. Offers special features such as "Activities" for duplication and classroom use; "Sharing Teaching Ideas" and "Applications." Reviews of instructional materials include computer software and more.

Journal for Research in Mathematics Education
This journal is published five times a year. A forum for disciplined inquiry into the teaching and learning of mathematics. Topics include reports of research, including experiments, case studies, surveys, philosophical studies, and historical studies. Also includes articles about research, including literature reviews and theoretical analyses, critiques of articles and books, and brief commentaries on issues pertaining to research.

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The Eisenhower National Clearinghouse for Mathematics and Science Education (ENC) is funded through the U. S. Department of Education to provide K–12 teachers with a central source of information on mathematics and science curriculum materials. ENC was established in 1992 through a contract with The Ohio State University and is located in Columbus, Ohio.

ENC encourages the adoption and use of K–12 curriculum materials and programs that support state and national efforts to improve teaching and learning in mathematics and science. It provides better access to resources by creating, maintaining, and cataloging a comprehensive, multimedia collection of materials and programs. The ENC catalog and other products are distributed nationally using both traditional formats and advanced computing and telecommunications technologies. Specifically, ENC provides the following products and services:

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■ There are twelve demonstration sites, located in conjunction with the 10 Eisenhower Regional Consortia, at The Capital Collection & Demonstration Site, and at ENC. Demonstration sites provide an opportunity for users to preview the ENC Online Information Service as well as a variety of software and other materials.

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For all Educators (please check all that apply):

*Years of K-12 teaching experience*_____ *Is your institution:* ❑ *Public* ❑ *Private*

Are you currently: ❑ *A classroom teacher* ❑ *A school department chair* ❑ *A curriculum specialist*
❑ *A school administrator* ❑ *A district administrator* ❑ *A librarian*
❑ *A teacher educator* ❑ *A college faculty member* ❑ *Other*_____

Is your area: ❑ *Science education* ❑ *Mathematics education* ❑ *Elementary education*
❑ *Some other area*_____

Teachers, please circle those grades which you teach
Administrators, circle those for which you have responsibility K 1 2 3 4 5 6 7 8 9 10 11 12
Teacher educators, circle those for which you prepare teachers

For K-12 Teachers and Administrators:
*Your school designation (e.g., Elementary, Middle, High, ...)*_____
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The National Council of Teachers of Mathematics

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